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THE INTERPRETATION OF FLYING QUALITIES REQUIREMENTS
FOR FLIGHT CONTROL SYSTEM DESIGN

E. G. Rynaski

TECHNICAL REPORT

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16. Abstract

A study was conducted to design an experimental flight test program for the Total In-Flight Simulator (TIFS) directed toward the interface between flying qualities requirements and flight control sytem design criteria. The eventual goal is to provide an interpretation or translation of flying qualities requirements for use by the flight control system designer. Specifically, an angle of attack and pitch rate command system matrix involving both short term and long term dynamics are specified for evaluation. A major objective of the research was to demonstrate that flying qualities criteria and flight control system configuration or architecture can be independent. Finally, additional configurations are proposed to evaluate the efficacy of dynamic decoupling.

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TABLE OF CONTENTS

TABLE OF CONTENTS (Cont'd)

LIST OF TABLES

LIST OF FIGURES

LIST OF SYMBOLS

$\delta(t)$ - Input or control vector, typically $\delta^T(t) = (\delta_e, \delta_T, \delta_z)$

$\delta_e(t)$ - moment effector input, typically elevator deflection

$\delta_{e_c}(t)$ - pilot stick command input

$\delta_z(t)$ - direct lift force effector input

$x(t)$ - State vector, typically $x^T(t) = (\dot{\theta}(t), \theta(t), \alpha(t), u(t))$

$\theta(t)$ - pitch attitude change from trim or reference

$\dot{\theta}(t) = q(t)$ - pitch rate

$\alpha(t)$ - angle of attack change

$w(t)$ - vertical velocity change from reference

$u(t)$ - change in speed from trim (body axis)

$\Delta V(t)$ - change in velocity from trim (wind axis)

$\gamma(t)$ - change in flight path angle

$n_z(t)$ - normal acceleration

F - Matrix of dimensional stability derivatives

G - Matrix of dimensional control derivatives

K - Matrix of gains

K_{ij} - a gain in the i^{th} row, j^{th} column of a gain matrix

ω_{sp} - Short period natural frequency

ω_p - Phugoid natural frequency

ω_α - Natural frequency of the complex transfer function zeros of the $\alpha/\delta_e(s)$ transfer function

ζ - Damping ratio

$-1/\tau_{\theta_2}$ - A zero of the $\theta/\delta_e(s)$ transfer function

$-1/\tau_{\theta_1}$ - A zero of the $\theta/\delta_e(s)$ transfer function

$-1/\tau_q$ - A zero associated with proportional plus integral pitch rate command/attitude hold system

$(\ )^T$ - Transpose of a vector or matrix

$(\ )^{-1}$ - Inverse of a matrix

M_i - Denotes a particular control system configuration

iff - If and only if

Section 1
INTRODUCTION

The flying qualities specification published in 1969, MIL-F-8785B(ASG) (Reference 1), for the first time quantitatively defined satisfactory and acceptable ranges of specific modal parameters of an aircraft. In particular, the acceptable frequencies and damping ratios or time constants of the short period and phugoid modes of longitudinal-vertical motions and the Dutch roll, roll mode and spiral modes of the lateral-directional degrees of freedom of motion were addressed.

The specification was the culmination of approximately twenty years of experimental flight research involving many variable stability aircraft, including the NT-33A, an F-94, B-26, Princeton Navion and Boeing 367-80. A wide range of aircraft types and sizes were therefore represented. For this representative range of aircraft types, the results were sufficiently consistent that it was generally not considered necessary to specify eigenvectors or, because the aircraft were assumed to have a conventional geometry and control surface complement, zeros of transfer functions. It was implicitly assumed that the modal residues would be naturally within limits associated with conventional and nominally stable aircraft configurations.

Three important developments among many in aircraft flight control technology have brought to light the necessity for continued fundamental research in flying qualities requirements. The first development was the appearance of dynamic elements of the control system in addition to the natural dynamics of the vehicle; this development increased the dynamic order of the system and thereby required some kind of definition of acceptability for these additional dynamic modes of motion. The second major development was the appearance, as personified by the F-16, of the statically unstable airplane configuration which generally required a considerable amount of feedback control to maintain stability and provide for satisfactory and acceptable flying qualities. Large loop gains generally produce large variations in pole-zero arrangements of transfer functions, thereby yielding a residue range in the modal responses considerably beyond the range obtained in conventional aircraft configurations or even those that use simple rate damping as augmentation. The third major development involves the use of additional means of producing forces and moments on the aircraft, such as canards, direct lift flaps and thrust vectoring. This development allowed for a total variability of vehicle dynamics involving not only poles but zeros of transfer functions as well. It becomes necessary to interpret these developments in terms of the existing flying qualities specification MIL-F-8785(C) (Reference 2) in order to provide guidelines for the flight control system designer. These guidelines are particularly important because the proposed MIL Standard and Handbook is slanted more toward the response of "axes" rather than a description of modal parameters (Reference 3). The purpose of the study documented in this report is to propose, with supporting rationale, a flight experiment on the USAF/AFWAL TIFS airplane that would help provide an interpretation of the MIL-F-8785(C) requirements for the flight control system designer.

Section 2

FLIGHT CONTROL DESIGN ORIENTATION

2.1 GENERAL DISCUSSION

The flight control system designer, whose primary task was to define control laws and other elements that would result in an airplane having satisfactory and acceptable flying qualities, was generally oriented towards the classical single input - single output control system viewpoint that the feedback quantity was the "controlled" variable. The tendency then was to require or desire the complete specification of one or a combination of response variables in terms of the completely defined transfer function(s) or step command time history. The classical approach to control system design, oriented towards a particular controlled quantity to have the attributes of smooth, relatively fast response to a step command input that can be generally characterized by a second order transfer function with no zeros in the transfer function. This control system design viewpoint is shared not only by those who would design the control system using classical methods such as root locus or Bode plots, but also by those who would use the modern control theory methods of linear optimal control, in which the response of the state variable(s) in the performance index approached that a Butterworth filter configuration; i.e., towards a response resembling an equivalent to a second order system with no transfer function zeros and a 0.707 damping ratio.

If the design philosophy described above is to be the design approach to be used for future flight control systems, it then becomes important to define the response variable(s) among many possibilities that would yield Level 1 vehicle flying qualities should that response variable exhibit the characteristic second order or second-order-like response whose transfer function contained no numerator zeros. A complicating development involves the preference among flight control system designers for particular sensors, such as a rate gyro, for inclusion in a feedback control law. The feedback of a particular response variable will tend to force the closed loop response of that variable toward a response characteristic of the response of a second or first-order-like transfer function with no numerator zeros. It is considered the "controlled" response variable.

Unquestionably, the most reliable and desirable instrument for inclusion in a feedback control law is the rate gyro. The recent emphasis in flight control system design therefore has been to use pitch rate feedback and to devise criteria based upon the pitch rate response of the vehicles without regard to whether the criteria is directly derived from MIL-F-8785(C) or even satisfies the intent of the flying qualities specification. When pitch rate feedback is used without compensation, the result will be a system that tends toward a first order response in pitch rate, indicating that the short period damping ratio has been made greater than critical. In addition, the phugoid poles tend toward the zeros at $-1/\tau_{\theta_1}$ and at the origin. In the limit a pole at the origin will produce a neutrally stable vehicle.

A proportional plus integral compensation network is often added to the feedforward path in series with the actuator. This additional pole increases the order of the system dynamics such that the resulting pitch rate response becomes dominantly second order as the loop gain is increased.

The basic dilemma in flying qualities then is to define the controlled response variable; i.e., the response of the airplane that should respond essentially as a first or second order system without zeros in the numerator of the transfer function. If this response variable is properly identified, then the desirable modal parameters such as short period natural frequency and damping ratio requirements of the MIL-F-8785(C) Standard should be directly applicable. Recent experiments (Reference 4, 5) suggest that the dominant emphasis in the MIL-F-8785C standard on the short term pitch rate response rather than angle of attack, $\dot{\gamma}$, or n_z may be inappropriate. A secondary dilemma involves control system design methods that use the most desirable sensor complement yet still satisfy the criteria.

2.2 INTERPRETATION OF FLYING QUALITIES REQUIREMENTS

A conventional airplane is an angle of attack or flight path rate commanded vehicle. The constant speed, short period approximation to the longitudinal-vertical equations of motion of an airplane yields an angle of attack transfer function that appropriates the form

$$\frac{\alpha}{\delta_F}(s) = \frac{M_\delta}{s^2 + 2\zeta_{sp}\omega_{sp}\, s + \omega_{sp}^2}$$

The dynamics of the short period angle of attack response are then completely characterized by the short period natural frequency and damping ratio. The transfer function is of the resulting "controlled variable" form. The phugoid or low frequency mode contains very little residue in the angle of attack response indicating that the phugoid poles are normally "close" to the numerator zeros of the transfer function. There is usually little low frequency oscillation or drift in the angle of attack response following a command input. It is not known whether these characteristics described above for the angle of attack response of the vehicle can be transfered or emulated by a pitch rate response with equal flying qualities goodness of results. This is not to say that if the results of the experiment point towards angle of attack or flight path as the appropriate "controlled variable" that pitch rate cannot be used as a feedback quantity. The specification of the appropriate "controlled variable" does not force the control system designer to use the "controlled quantity" as a feedback variable.

2.3 CONFIGURATION DEFINITIONS

The orientation of the control system designer is to specify a control system as commanding a particular response variable. In the short term response of an airplane, the commanded variable could be either or a combination of the states pitch rate or angle of attack or an output quantity such as vertical acceleration. To which response variable should the flying qualities specification apply? An experiment is described below that is designed to answer this question, thereby providing the control system designer with an essential guideline on how to interpret the flying qualities specification. It is not clearly indicated in the MIL-F-8785(C) standard whether the pitch axis response should apply to pitch rate or angle of attack or to some combination of the states.

This question is a very important one. The tendency has been to concentrate on pitch and often only on pitch, yet there is at least equal reason to believe that the application of the specification to angle of attack or flight path angle response is even more appropriate. It is the purpose of this report and task plan to outline an experiment that would go a long way towards defining the appropriate controlled variable for the particular task of approach and landing.

The experiment described below considers an angle of attack and a pitch rate command system in terms of the locations of the poles of the system with respect to the zeros in the pitch rate and angle of attack transfer functions. In this respect, the intent is to try to determine whether the modal parameter approach as specified in MIL-F-8785(B) has been properly interpreted in the MIL-F-8785(C) standard in terms of aircraft axes. The short period and the phugoid will be considered separately because it is possible to design a pitch rate command system for the short term, but an angle of attack command system in the long term, or vice versa. The idea is to try to determine pilot preference both in the short term and long term.

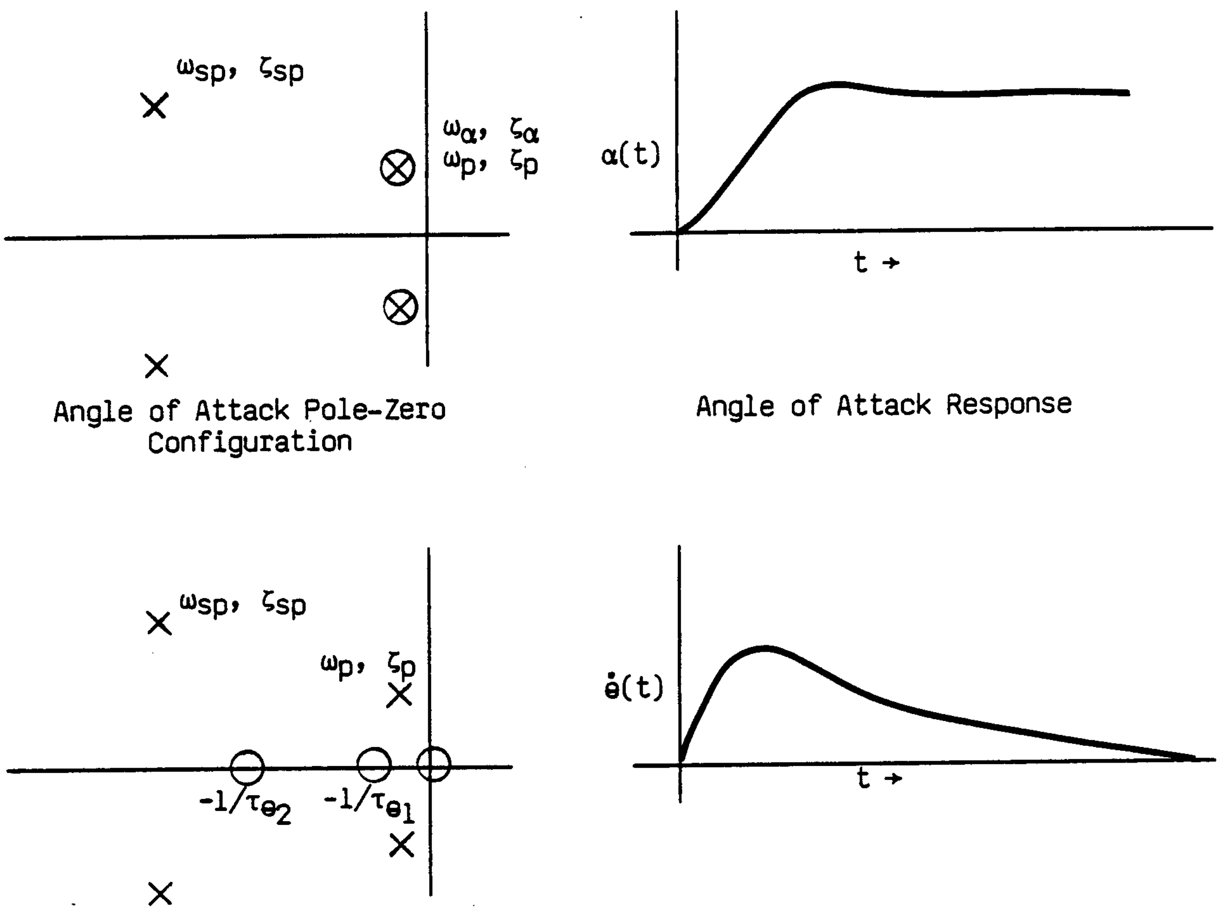
The angle of attack or the pitch rate command systems can be defined solely in terms of the locations of the vehicle poles with respect to the zeros of the transfer functions. A pitch rate command, attitude hold system will produce a pole-zero cancellation such that three poles are placed at the zeros of the transfer function located at the origin of the s plane, at $-1/\tau_{\theta_1}$, and at $-1/\tau_{\theta_2}$. Therefore, the response in pitch rate is dominated by the one remaining pole. In the angle of attack command system two poles are located at the low frequency zeros ζ_α, ω_α of the angle of attack transfer function. The response is dominated by the remaining two poles, which define the short period response. These systems are briefly described below without regard to how they may be mechanized. The mechanization problem is not a difficult one and will be discussed in a later section of the report.

The sketches below indicate the pole-zero patterns that are representative of the different "controlled variable" configurations described above.

2.3.1 Angle of Attack Command System

Pole-Zero Configuration

Response to Step Command



Angle of Attack Pole-Zero Configuration

Angle of Attack Response

Pitch Rate Pole-Zero Configuration

Pitch Rate Response

Figure 1. **ANGLE OF ATTACK COMMAND SYSTEM**

As shown in Figure 1, the response of the angle of attack command system is dominated by the short period poles ω_{sp}, ζ_{sp}. The phugoid poles are located at the low frequency zeros ω_α, ζ_α of the $\alpha/\delta_F(s)$ transfer function. The result can be a quick, smooth and well behaved angle of attack response as defined by the short period mode. Theoretically there is no residue in the angle of attack response in the phugoid mode; i.e., $\dot{\alpha}(t) = 0$ after the short period response.

The pitch rate response of the angle of attack command system is typical of a conventional aircraft. The transfer function zero at $-1/\tau_{\theta 2}$ produces an overshoot in the pitch rate response to a step command input, and a significant phugoid mode residue with zero ultimate steady state value is evident.

2.3.2 Pitch Rate Command System

As the angle of attack command system showed pole-zero cancellation in the angle of attack transfer function the pitch rate command system indicates pole-zero cancellation in the pitch rate command system. The pole-zero pattern showing these cancellations are displayed in Figure 2 below.

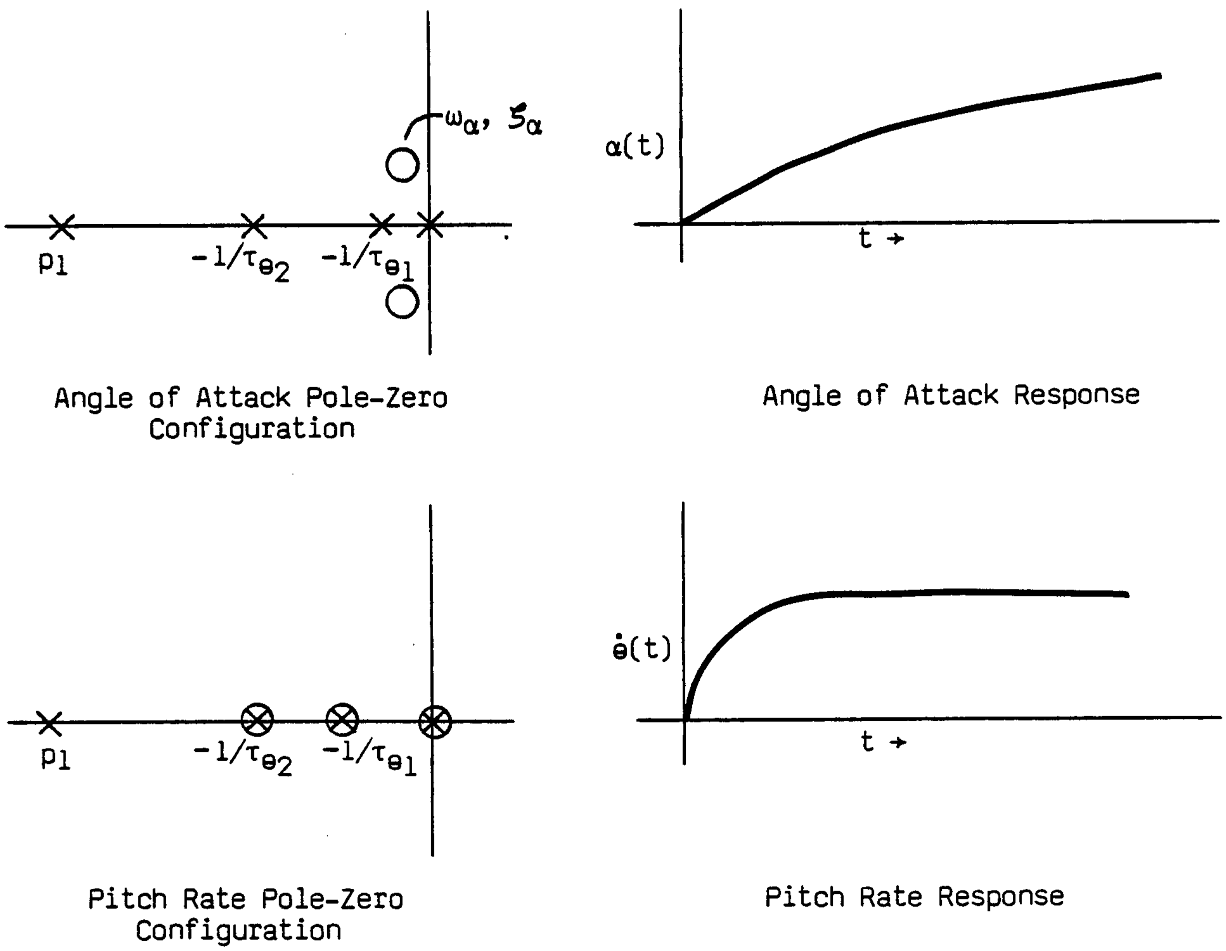


Figure 2. PITCH RATE COMMAND SYSTEM

The response of the pitch rate command system is dominated by the single order pole p_1 shown in the above figure. The response shows no residue in the phugoid mode, and the zero at the origin is cancelled by a pole, which indicates that the system will be an attitude hold system. The angle of attack response is generally sluggish, dominated by the poles at $-1/\tau_{\theta_2}$ and $-1/\tau_{\theta_1}$ that are not cancelled by numerator zeros of the angle of attack transfer function. The pole at the origin also contributes to the response and leads to a steady state ramp response in angle of attack. After the step input is returned to zero, the pitch rate returns to zero but the change in angle of attack does not. The pitch rate command, attitude hold system is also an angle of attack "hold" system, although the response in angle of attack is normally so sluggish that steady state angle of attack would likely be rarely seen in actual flight. Speed change will also exhibit neutral stability.

The two types of system described in Figure 1 and Figure 2, mainly a pitch rate command and angle of attack command system involved both short period and phugoid dynamic behavior of both of the response variables and each part contributed significantly to the dynamic behavior of the system. The low frequency behavior of the angle of attack command system is such that after the angle of attack reaches steady state, then changes in flight path are equal to changes in pitch angle; i.e., $\Delta\gamma=\Delta\theta$ since $\dot{\alpha}(t)=0$. The pilot can judge changes in flight path by observing changes in pitch angle. Because pitch rate eventually goes to zero following a step command, the pitch attitude and the flight path reach new steady state values. In the pitch rate command, attitude hold system, the angle of attack responds sluggishly and never reaches a steady state value to a step command input. The change in flight path angle is not equal to changes in pitch angle; i.e., $\Delta\theta \neq \Delta\gamma$ and the pilot has more difficulty in judging changes in flight path by observation of changes in pitch angle. The result of the sluggish angle of attack response is frequently an overcontrol tendency by the pilot during flare and landing. A correction of the overcontrol leads to pilot complaints of "non-monotonic" stick forces.

The differences in the short period response are more obvious. In the angle of attack command system, the numerator zero in the pitch rate transfer function may be considered a lead term in the pitch rate response. In the pitch rate command system, the singularity that previously was a pitch rate lead becomes a pole or lag in the angle of attack response.

2.3.3 Hybrid Systems

Simple variations in the types of "pure" controlled variable systems should allow both the flying qualities engineer and the flight control system designer to determine whether or not the "controlled variable" philosophy of control system design applies to both the short term and the long term or phugoid mode. For instance, by the independent placement of the short period and phugoid poles it is a relatively simple matter to obtain a short term angle of attack command, long term pitch rate command system. This can be done as shown in Figure 3 below, in which the short period poles are placed as if the system were angle of attack command, while the phugoid poles are placed as if the system were pitch rate command. The converse, as shown in Figure 4 below, can also be accurately evaluated using an aircraft such as the USAF/Calspan Total In-Flight Simulator (TIFS).

In the past it has been often stated that the pilot is little affected by the long term or phugoid motion of the vehicle. It has been assumed that the pilot either ignores these long term effects or corrects for them more-or-less subconsciously. If this hypothesis is true, then it should make no difference if the phugoid poles were located at either the zeros of the numerator of the angle of attack transfer function (ω_α, ζ_α) or at the origin and at $-1/\tau_{\theta_1}$, two of the numerator zeros of the pitch rate transfer function. It is expected that the hybrid variations depicted by Figures 3 and 4 should help significantly to settle the question of the importance of phugoid dynamics with respect to flying qualities.

2.3.3.1 Short Term Angle of Attack - Attitude Hold

The short term angle of attack, long term pitch rate command system pole-zero configuration is shown in Figure 3 below.

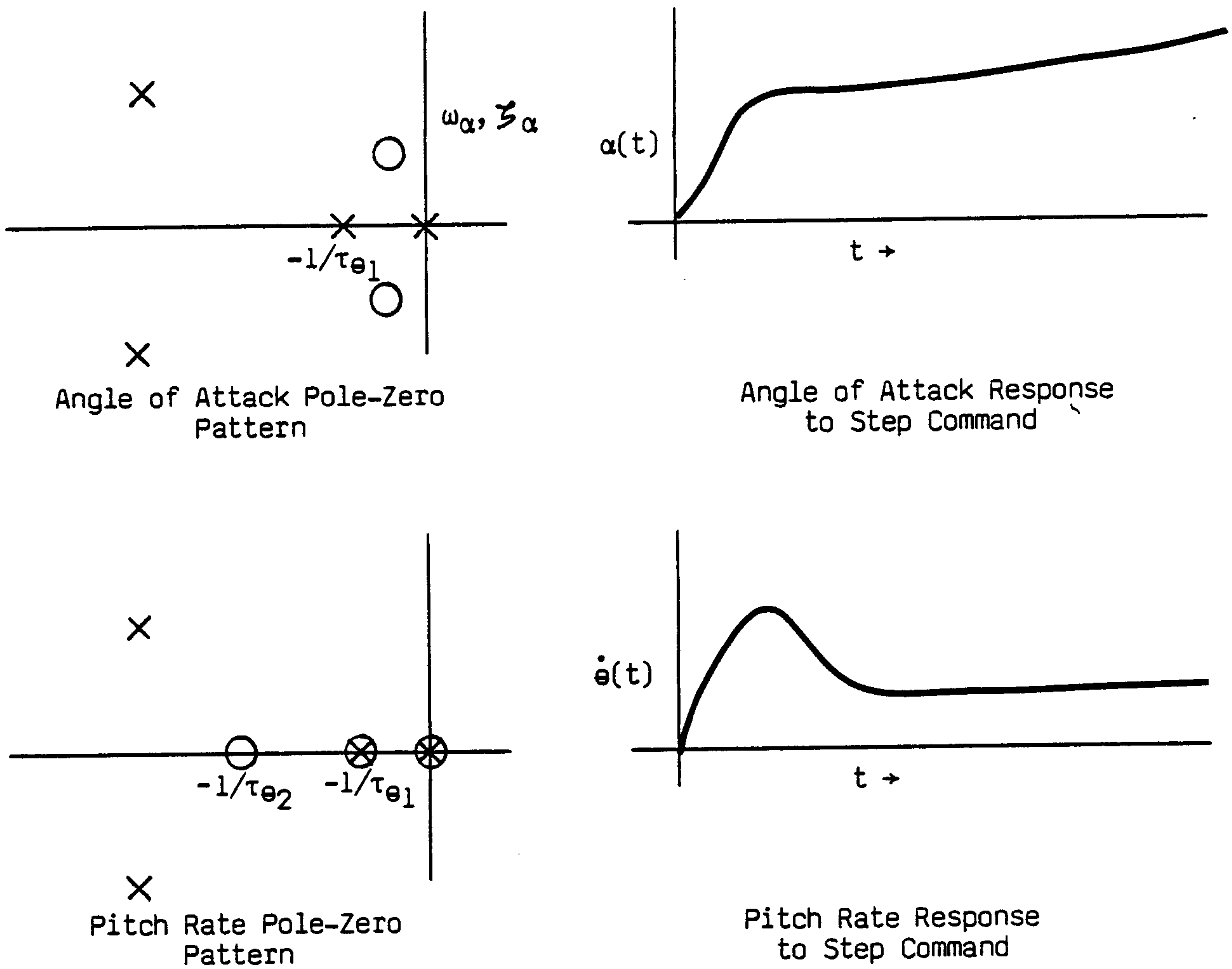


Figure 3. **SHORT TERM ANGLE OF ATTACK, LONG TERM PITCH RATE COMMAND SYSTEM**

The behavior of this system is characterized by the smooth and rapid short period angle of attack response and the modal residues of the poles located at $1/\tau_{\theta_1}$ and at the origin. The pitch rate response is characterized by an initial pitch rate overshoot followed by a steady state pitching rate; no phugoid mode residue is evident.

2.3.3.2 Short Term Pitch Rate - Long Term Angle of Attack Command System

The short term pitch rate, long term angle of attack system is shown in Figure 4 below:

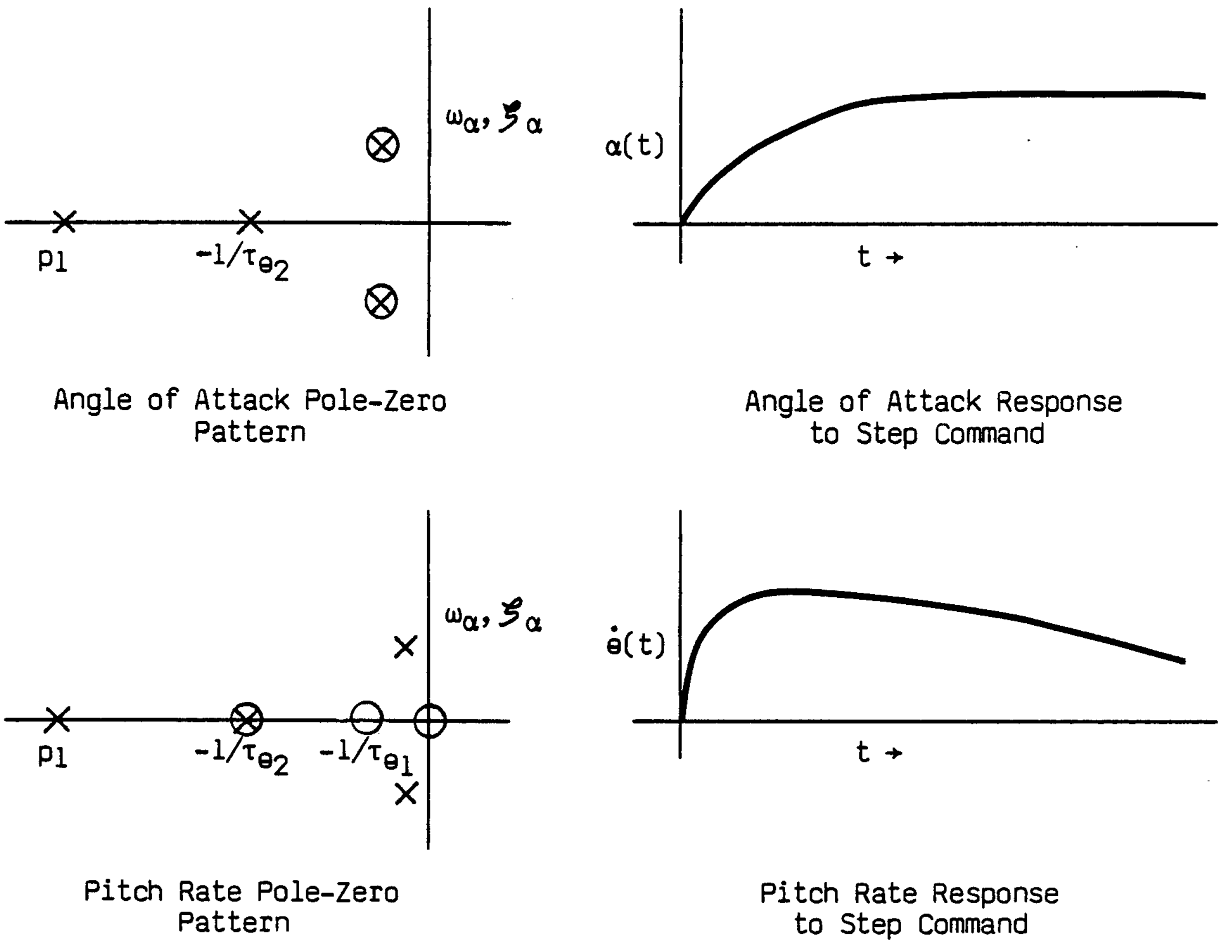
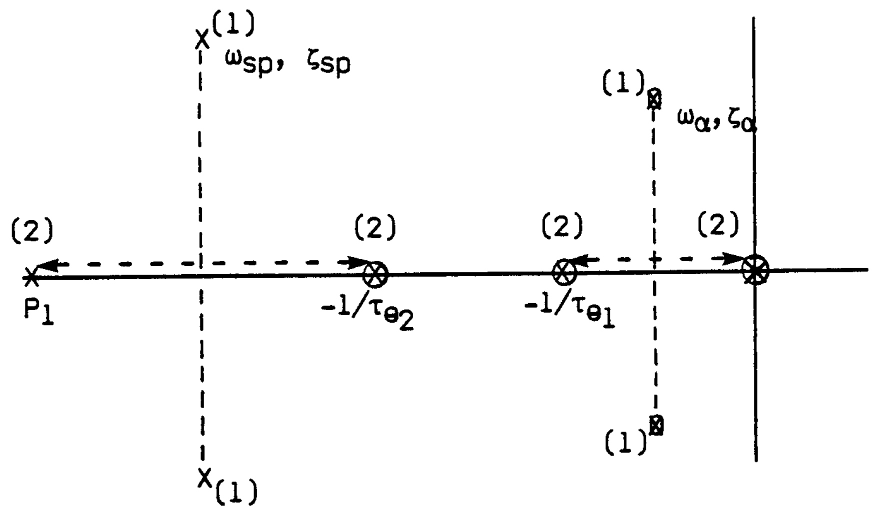


Figure 4. SHORT TERM PITCH RATE, LONG TERM ANGLE OF ATTACK COMMAND SYSTEM

The behavior of this system is characterized by an angle of attack response dominated by the pole at $-1/\tau_{\theta_2}$ and can be sluggish. The angle of attack response remains steady in the long term. The pitch rate response is initially rapid and dominated by the single pole at $-p_1$, but then exhibits the effects of a significant residue at the phugoid mode frequency.

2.3.4 Range of Pole Requirements

An examination of the four configurations of Figures 1 thru 4 above show that the difference between a pitch rate and an angle of attack or path command system is a matter of pole placement with respect to the existing and fixed zeros of the transfer functions. The range of pole variations for angle of attack and pitch rate command are sketched in Figure 5.



(1) Pole locations for
$\alpha(t)$ command system

(2) Pole locations for
$\dot{\theta}$ command system

Figure 5. RANGE OF POLES THAT RESULT IN PITCH RATE OR ANGLE OF ATTACK COMMAND SYSTEMS

Most of the resonable variations or trade-offs between an angle of attack command system and a pitch rate command system are shown by the dotted lines of Figure 5. There is every reason to believe that satisfactory and acceptable flying qualities might be obtained at many points along the loci shown above, indicating that a weighted combination or compromise between a pitch rate command and angle of attack command system could be optimum for a particular piloting task.

2.3.5 Multi-controller Design

Feedback from only one controller can alter only the closed loop poles of the system. Feedback using two controllers, such as an elevator and direct lift flap, can be used to alter not only the poles, but the closed loop zeros of the individual transfer functions as well. For instance, feedback to a second controller can be used to force low frequency transfer function zeros in both the angle of attack and the pitch rate transfer functions to be identical. A system of this type could be characterized by the simplified transfer functions defined below:

$$\frac{\dot{\theta}}{\delta}(s) = \frac{K_1(s + 1/\tau_{\theta_2})}{s^2 + 2\zeta_{sp}\omega_{sp}s + \omega_{sp}^2} \tag{2-1}$$

$$\frac{\alpha}{\delta}(s) = \frac{K_2}{s^2 + 2\zeta_{sp}\omega_{sp}s + \omega_{sp}^2} \tag{2-2}$$

$$\frac{\Delta V}{\delta}(s) = \frac{K_3(s + 1/T_1)(s + 1/T_2)}{\left[s^2 + 2\zeta_{sp}\omega_{sp}s + \omega_{sp}^2\right]\left[s^2 + 2\zeta_p\omega_p s + \omega_p^2\right]} \tag{2-3}$$

The type of system described by Equation 2-1 through 2-3 would be an angle of attack or pitch rate command system in the short term response depending on the short period frequency and damping ratio selected and <u>both</u> a pitch rate and angle of attack command system with respect to the long term or phugoid mode of response. In other words, no residue of the phugoid mode would appear in either the angle of attack or the pitch rate response of the system. This type of vehicle behavior could guarantee that the system would exhibit both an attitude and flight path hold behavior, with $\Delta\gamma = \Delta\theta$ at all times after the short term response, and with attitude and flight path precisely controllable by the pilot. The phugoid mode would appear only in the speed change degree of freedom of motion.

2.4 EXAMPLES

Examples of the pitch rate command and angle of attack or flight path command system configurations were computed to illustrate the dynamic behavior of these types of systems.

The examples were designed to illustrate several important features of the different types of command systems possible. The first characteristic was to demonstrate the different responses for pitch rate and angle of attack or flight path command systems. The second important element affecting the flying qualities is the behavior of the long term or phugoid dynamics. With respect to the phugoid dynamics alone, the system can be pitch rate or angle of attack command, independent of the short period. Finally, the effect of varying both $1/\tau_{\theta_2}$ and the phugoid frequency and damping ratio are illustrated. As can be seen from the plots, the phugoid frequency and damping ratio has a strong effect on the response of the system.

The examples shown in Table 1 below exhibit the following characteristics.

1. Group I configurations were computed with a value of $1/\tau_{\theta_2} = 0.5$. The short period natural frequency and damping ratio were chosen to be $\omega_{sp} = 2$ rad/sec $\zeta = 0.7$ for the angle of attack command systems. The phugoid frequency and damping ratio were chosen to be equal to the lightly damped low frequency zeros of the angle of attack transfer function. For the pitch rate command system, the short period poles were located at $p_1 = -1/\tau_{\theta_2}$, $p_2 = -\tau_{\theta_2}\,\omega_{sp}^2$, with $\omega_{sp} = 2$ rad/sec. Therefore, $p_1 p_2 = \omega_{sp}^2$. The phugoid poles were chosen to be located at the origin and at $-1/\tau_{\theta_1}$.

2. Two changes were made in the Group II configurations as compared to Group I. For group II the phugoid frequency was changed from $\omega_p = 0.2$ to $\omega_p = 0.1$. In addition, the value of $1/\tau_{\theta_2}$ was changed from $1/\tau_{\theta_2} = 0.5$ to $1/\tau_{\theta_2} = 0.9$. The $1/\tau_{\theta_2}$ change produced a short period damping ratio of about 1.3, within the Level 1 damping ratio requirements of MIL-F-8785(C).

The transfer functions were chosen in all cases to be as defined below

$$\frac{\dot{\theta}}{\delta_F}(s) = \frac{-20s(s + 1/\tau_{\theta_1})(s + 1/\tau_{\theta_2})}{D_i(s)}$$

$$\frac{\alpha}{\delta_F}(s) = \frac{-1.8(s + 10)\left[s^2 + 2\zeta_\alpha\,\omega_\alpha\,s + \omega_\alpha{}^2\right]}{D_i(s)}$$

$$\frac{\Delta V}{\delta_F}(s) = \frac{-25(s + 1)(s - 15)}{D_i(s)}$$

The denominator polynomials $D_i(s)$ are defined as

1. Pitch Rate Command

$$D_1(s) = s(s + 1/\tau_{\theta_1})(s + 1/\tau_{\theta_2})(s + p_2)$$

2. Angle of Attack Command

$$D_2(s) = \left[s^2 + 2\zeta_{sp}\,\omega_{sp}\,s + \omega_{sp}{}^2\right]\left[s^2 + 2\zeta_\alpha\,\omega_\alpha\,s + \omega_\alpha{}^2\right]$$

3. Short Term Angle of Attack Command/Attitude Hold

$$D_3(s) = s(s + 1/\tau_{\theta_1})\left[s^2 + 2\zeta_{sp}\,\omega_{sp}\,s + \omega_{sp}{}^2\right]$$

4. Short Term Pitch Rate Command/Long Term Angle of Attack Command

$$D_4(s) = (s + 1/\tau_{\theta_2})(s + p_2)\left[s^2 + 2\zeta_\alpha\,\omega_\alpha\,s + \omega_\alpha{}^2\right]$$

Table 1

RATE AND PATH COMMAND CONFIGURATIONS

SYSTEM	FIG	CONFIG.	ω_{sp}(rad/sec)	ζ_{sp}	ω_{ph}(rad/sec)	ζ_{ph}	$1/\tau_{\theta_2}$
1	6	I-A	p_1 = -.500	p_2 = -8.0	p_3 = -0.10	p_4 = 0.0	0.50
2	7	I-B	2.00	0.70	0.20	0.10	0.50
3	8	I-C	2.00	0.70	p_3 = -0.10	p_4 = 0.0	0.50
4	9	I-D	p_1 = -.500	p_2 = -8.00	0.20	0.10	0.50
1	10	II-A	p_1 = -0.90	p_2 = -4.40	p_3 = -0.10	p_4 = 0.0	0.90
2	11	II-B	2.00	0.70	0.10	0.10	0.90
3	12	II-C	2.00	0.70	p_3 = -0.10	p_4 = 0.0	0.90
4	13	II-D	p_1 = -0.90	p_2 = -4.40	0.10	0.10	0.90

Note: $1/\tau_{\theta_1}$ = 0.10 for all cases

Discussion

Figures 6 through 9 show the responses to a long, ten second pulse for the Group I system, while Figures 10 through 13, Group II, show the effects of altering $1/\tau_{\theta_2}$ and the phugoid mode frequency. Figures 6 and 10 show the pitch rate command/attitude hold system as can be seen by the pitch rate and pitch attitude responses of Figure 6a, b and 10a, b. The pitch rate response is dominated by the poles at $p_2 = -8.0$ in Figure 6 and $p_2 = -4.40$ in Figure 10 and are similar in shape. The angle of attack responses, however, are considerably different, as shown in Figure 6c and 10c. The angle of attack response of Configuration Ia, shown in Figure 6c, appears to show a dominant residue in the pole at $p = -1/\tau_{\theta_1}$. The angle of attack drifts off, which means that after the short term response, $\Delta\theta \neq \Delta\gamma$ and the pilot may have difficulty judging changes in flight path by observing changes in pitch attitude. Figure 10c shows the angle of attack response to be dominated by the pole at $p = -1/\tau_{\theta_2}$ with much smaller residue in the pole at $-1/\tau_{\theta_1}$. The effect is to produce a system that responds rapidly and smoothly in both pitch rate and in angle of attack, so that in the long term $\Delta\theta \cong \Delta\gamma$. This comparison between Figure 6 and Figure 10 illustrates the effect of changes in phugoid frequency and $1/\tau_{\theta_2}$.

Figures 7 and 11 demonstrate the the system response when configured as an angle of attack command system, both in the short term and long term modes of response of the vehicle. Figure 7a shows the characteristic pitch rate overshoot and the effects of the phugoid mode that enters into the response rather rapidly. Figure 11a shows the effect of a lower phugoid frequency, which results in a greater seperation in frequency between the short period and phugoid modes and much less phugoid residue in the pitch response. It may be that phugoid-short period separation is more important than values of $1/\tau_{\theta_2}$. As shown by Figure 7c and 11c, the angle of attack responses are identical with zero phugoid mode residue. Therefore, the angle of attack remains constant following the short period response and $\Delta\gamma = \Delta\theta$. As can be seen by comparing the pitch attitude behavior in Figure 7 and 11, the attitude and the flight path response of Figure 11 remains much more constant and predictable after the step input is removed. It appears that an important variable of this experiment could be the separation between the short period and the phugoid mode frequencies or the relationship between the phugoid frequency and the value of $1/\tau_{\theta_1}$ or both.

The short term angle of attack command - attitude hold system responses are shown in Figures 8 and 12. The systems are categorized by pitch rate overshoot in the short term and zero phugoid residue in the long term. In each case, the response is attitude hold, as shown by Figure 8b and 12b after the step input command has been removed. Figures 8c and 12c show the resulting angle of attack responses; smooth, fast and well behaved in the short term but with significant long term modal response residues. The smaller residues of the low frequency portion of the response in Figure 12c might likely be attributable to the larger value of $1/\tau_{\theta_2}$. The effect is to produce a system in which $\Delta\gamma$ is proportional to $\Delta\theta$ in the long term, a characteristic deemed to be desirable.

The short term pitch rate command - long term angle of attack command system responses are shown in Figure 9 and 13. The pitch rate responses of Figure 9a shows the rapid pitch rate response due to the pole at $p = -8.0$ but also shows a very large low frequency mode residue in the response. The much lower low frequency residue in the responses of Figure 13 indicate an improved ability to not only point the aircraft, but have the aircraft fly in the direction in which it is being pointed.

Commentary

If it is assumed that a rapid flight path response is desirable along with an ability to maintain attitude and flight path angle correspondence following the short term response, it would appear that the system with a higher value of $1/\tau_{\theta_2}$ and a lower phugoid frequency is superior. The larger separation in short period and phugoid frequencies slows down the tendency of the response variables to "drift" or show significant residue after the initial, short term vehicle response. The result is an improved tendency to maintain attitude and flight path angle correspondence following the short period response, for all four types of system. It might appear that both pitch rate command attitude hold and angle of attack command systems can have favorable flying qualities if the system dynamics are properly configured. For single controller aircraft, only the poles can be altered with feedback and placed anywhere. Therefore, the important parameters that will determine whether or not a particular aircraft can be made to have level 1 flying qualities for a particular type of "command" system are likely to be the values of the transfer function zeros.

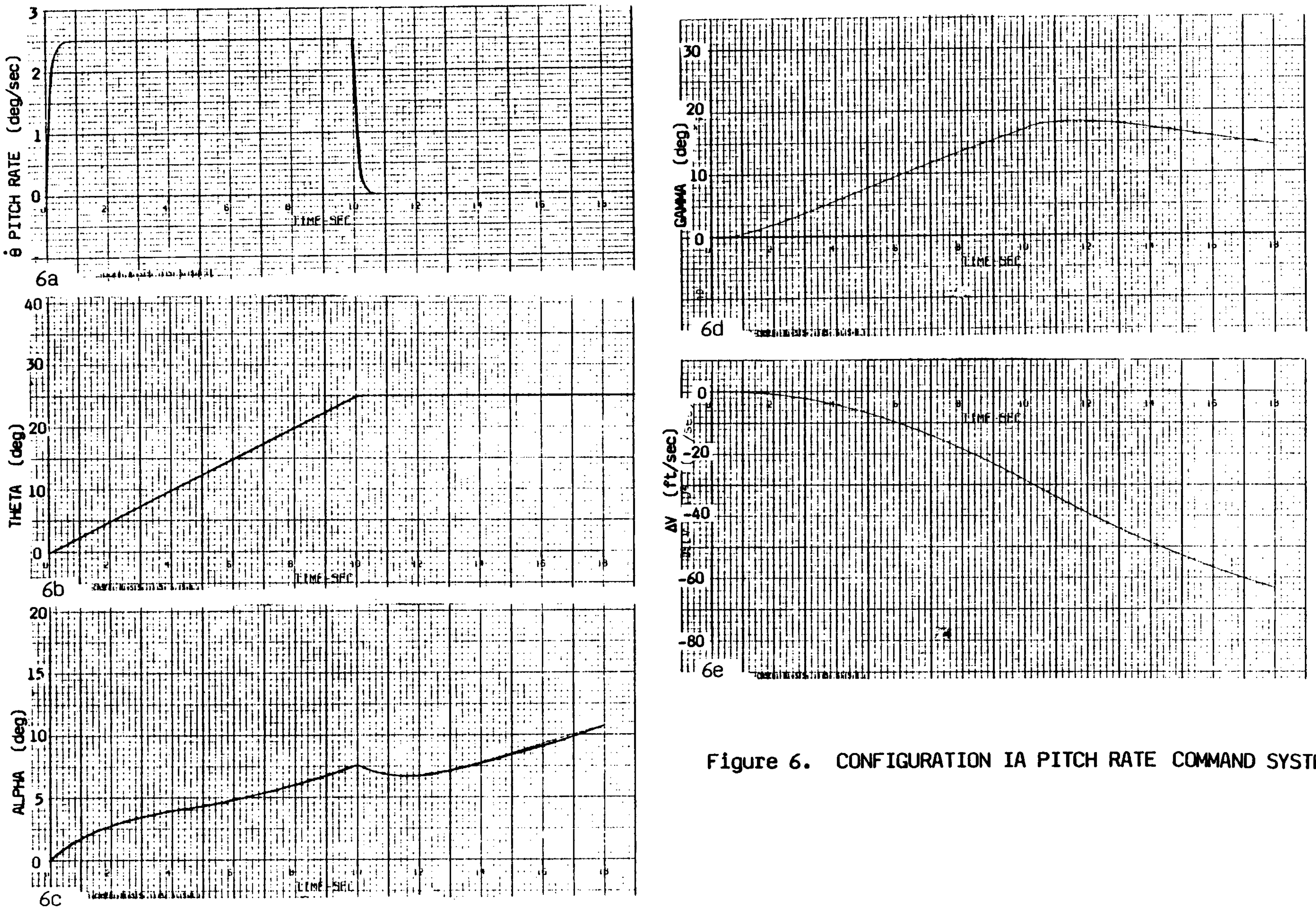


Figure 6. CONFIGURATION IA PITCH RATE COMMAND SYSTEM

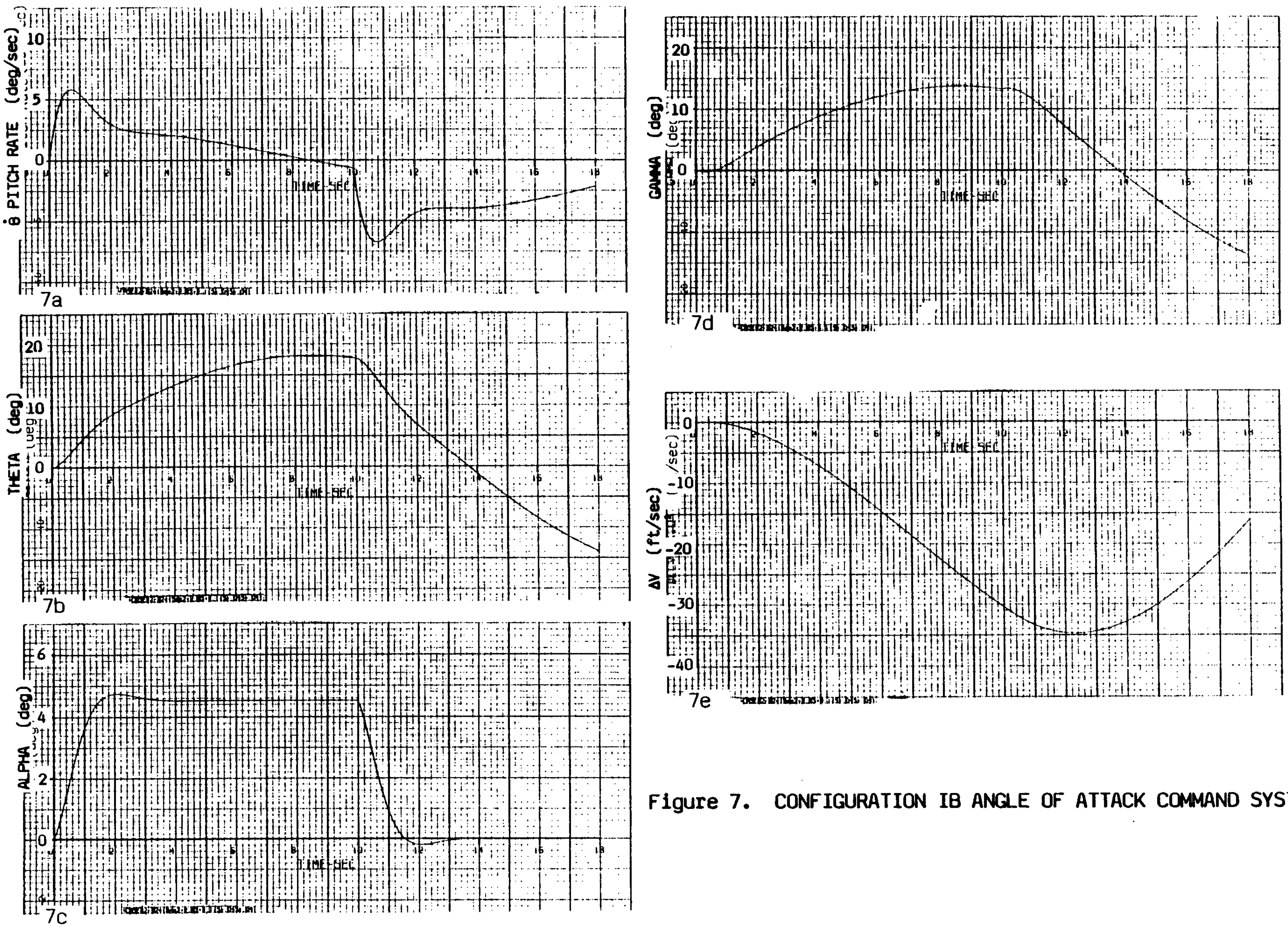


Figure 7. CONFIGURATION IB ANGLE OF ATTACK COMMAND SYSTEM

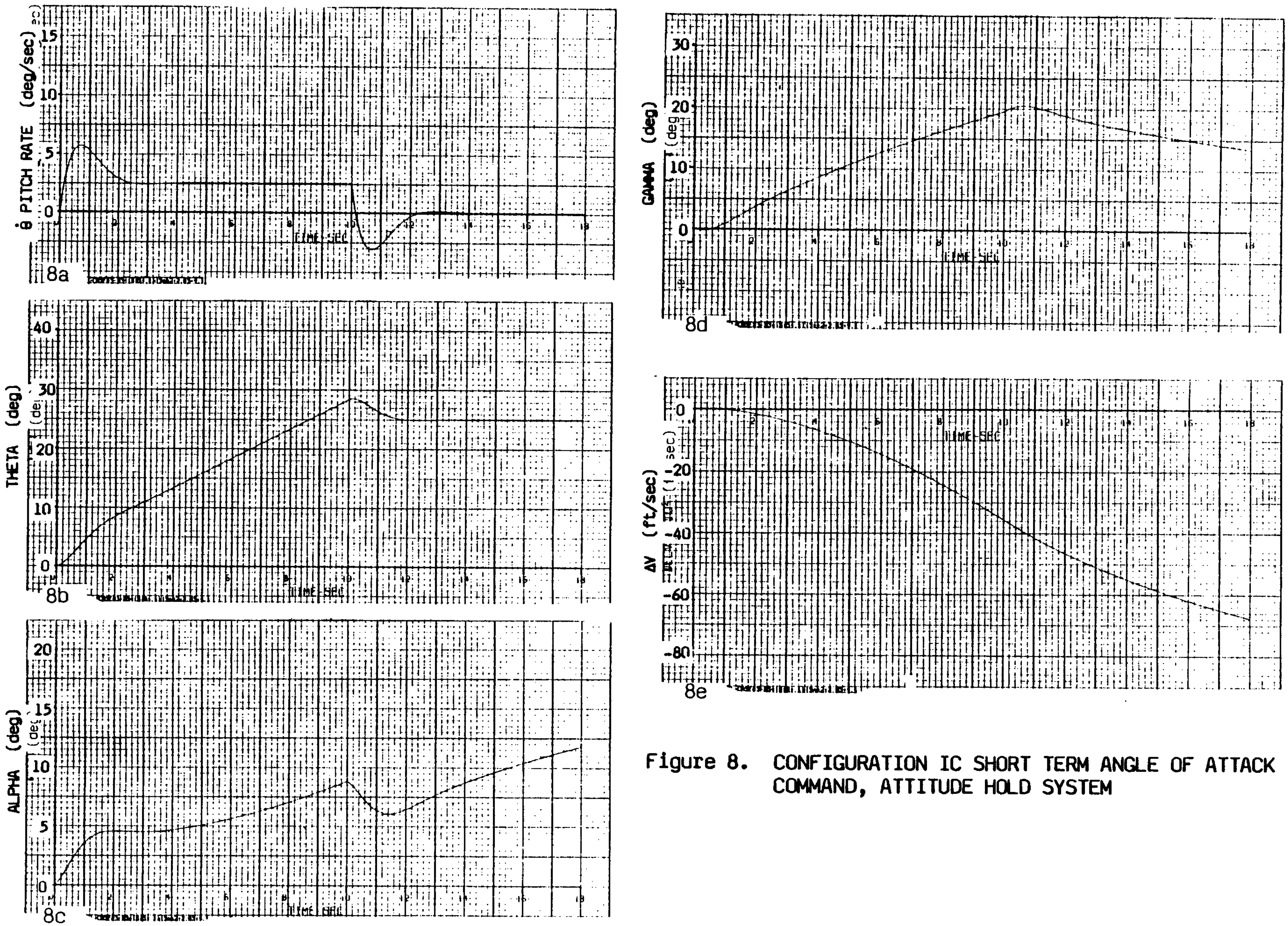


Figure 8. CONFIGURATION IC SHORT TERM ANGLE OF ATTACK COMMAND, ATTITUDE HOLD SYSTEM

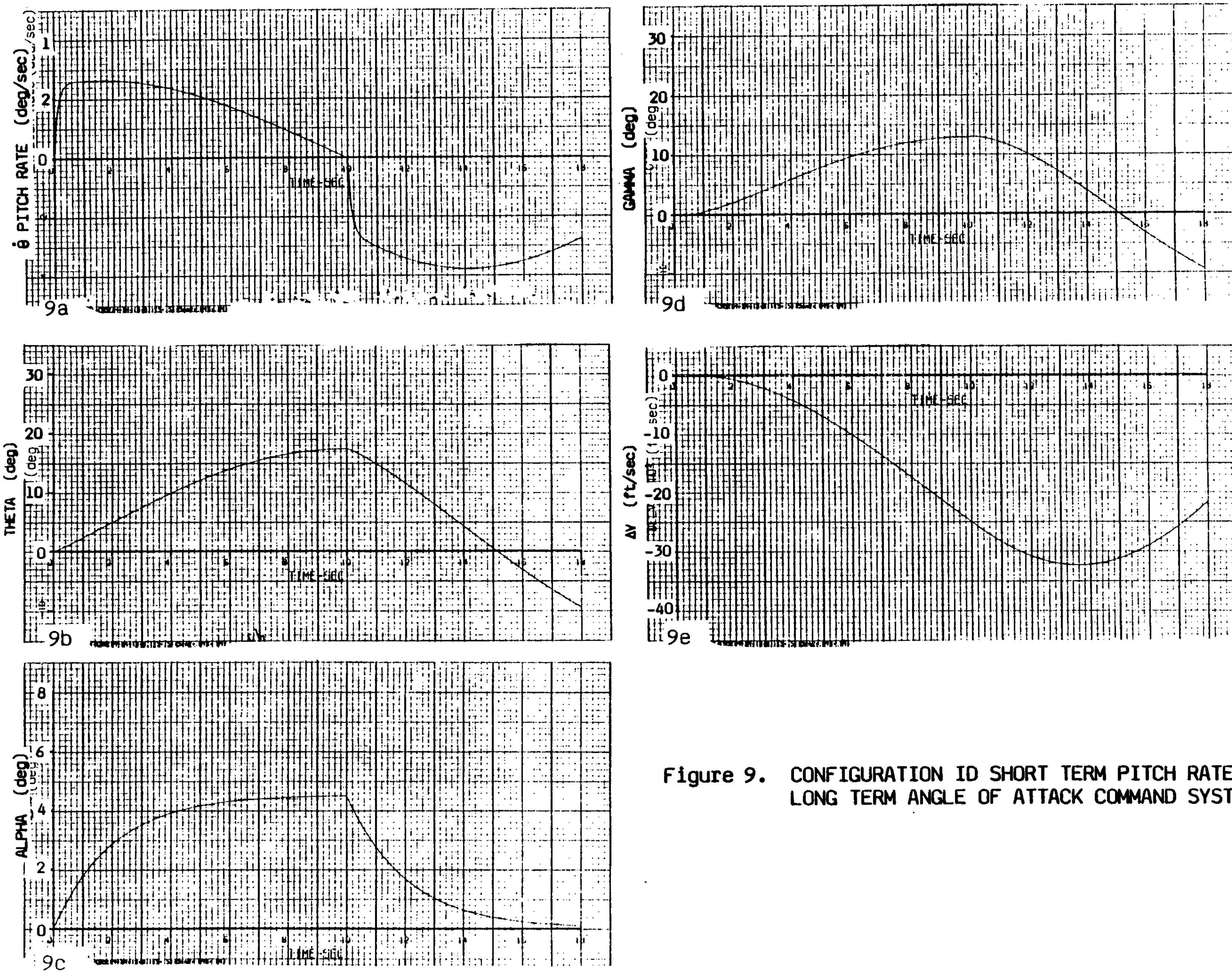


Figure 9. CONFIGURATION ID SHORT TERM PITCH RATE, LONG TERM ANGLE OF ATTACK COMMAND SYSTEM

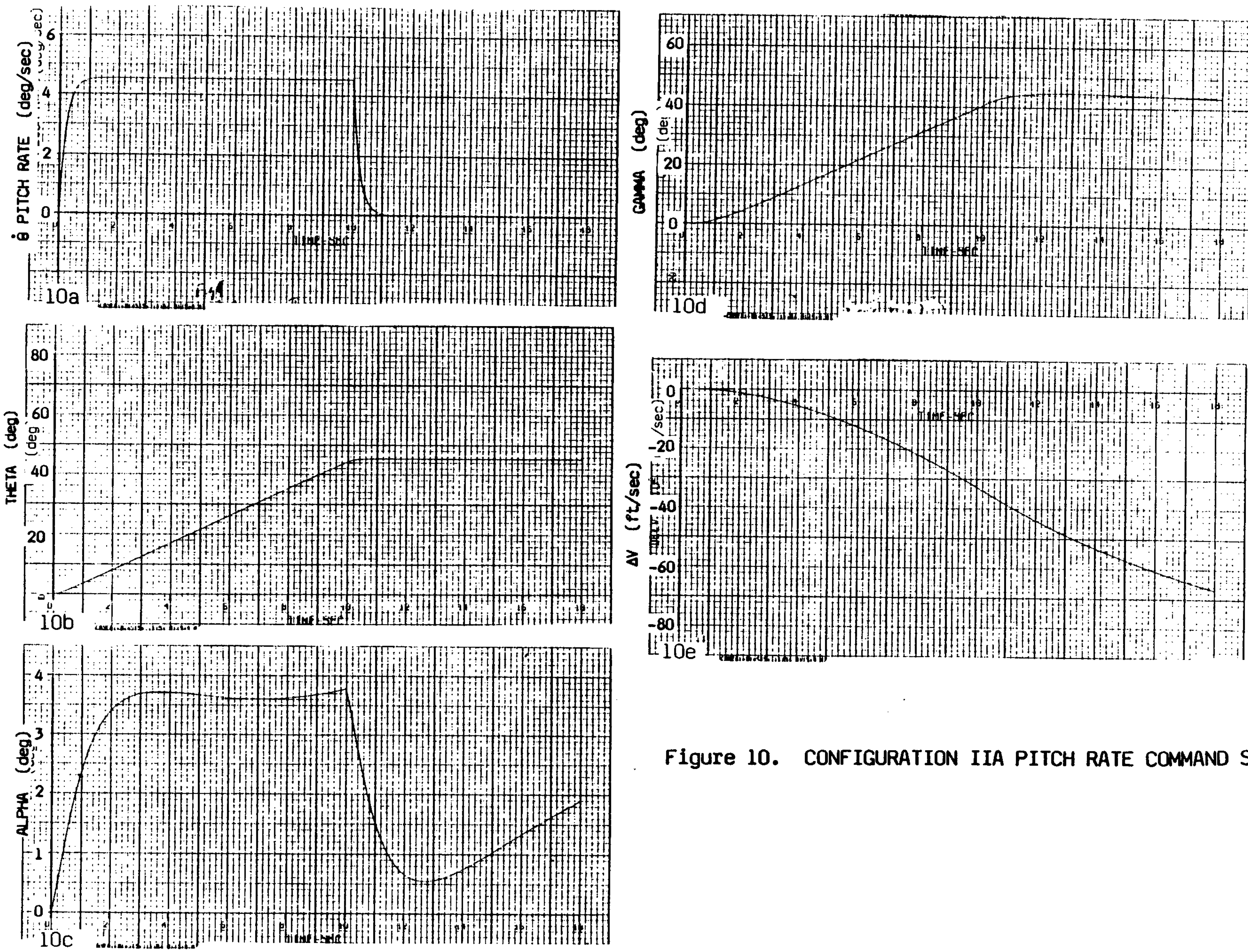


Figure 10. CONFIGURATION IIA PITCH RATE COMMAND SYSTEM

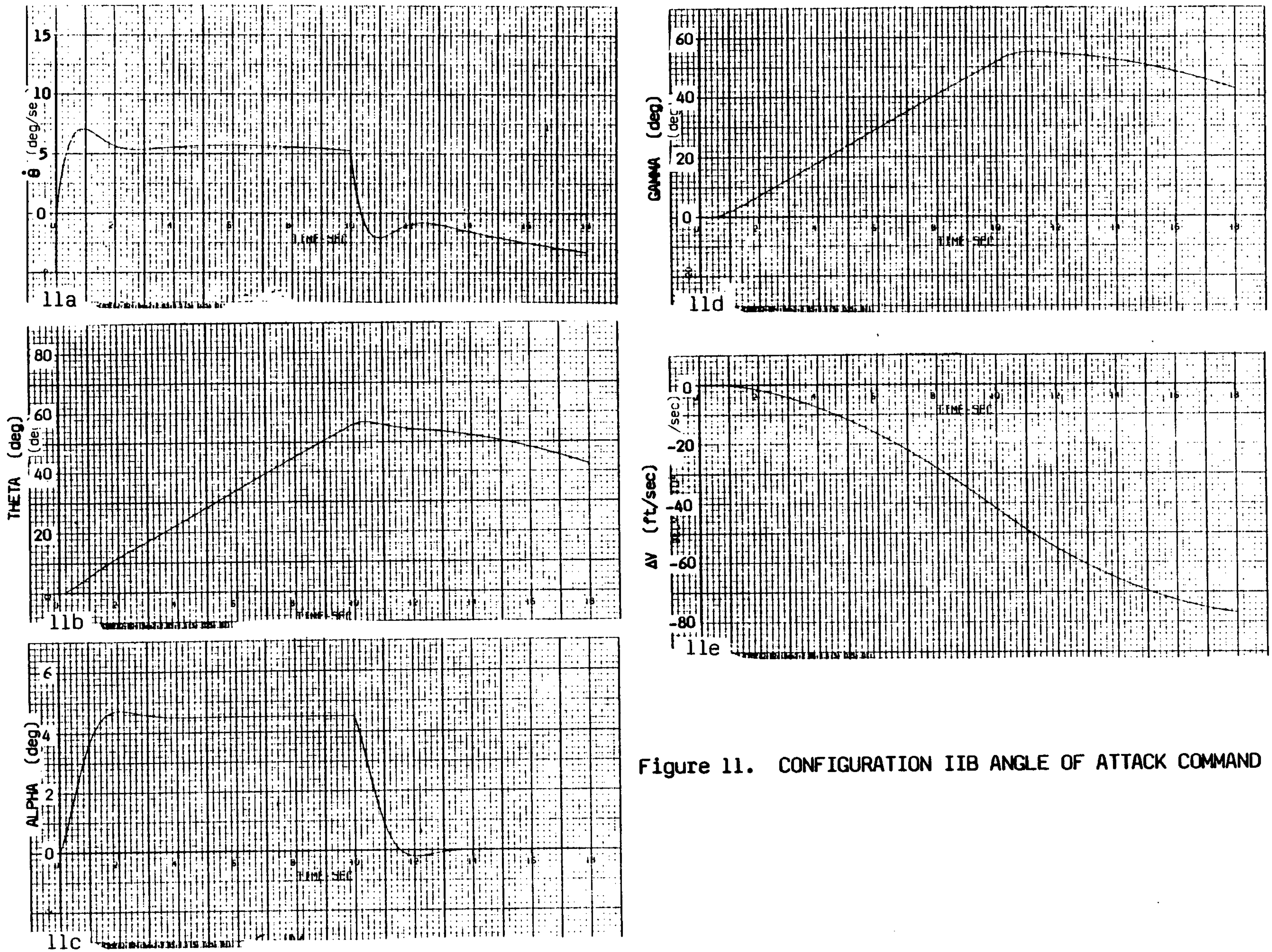


Figure 11. CONFIGURATION IIB ANGLE OF ATTACK COMMAND SYSTEM

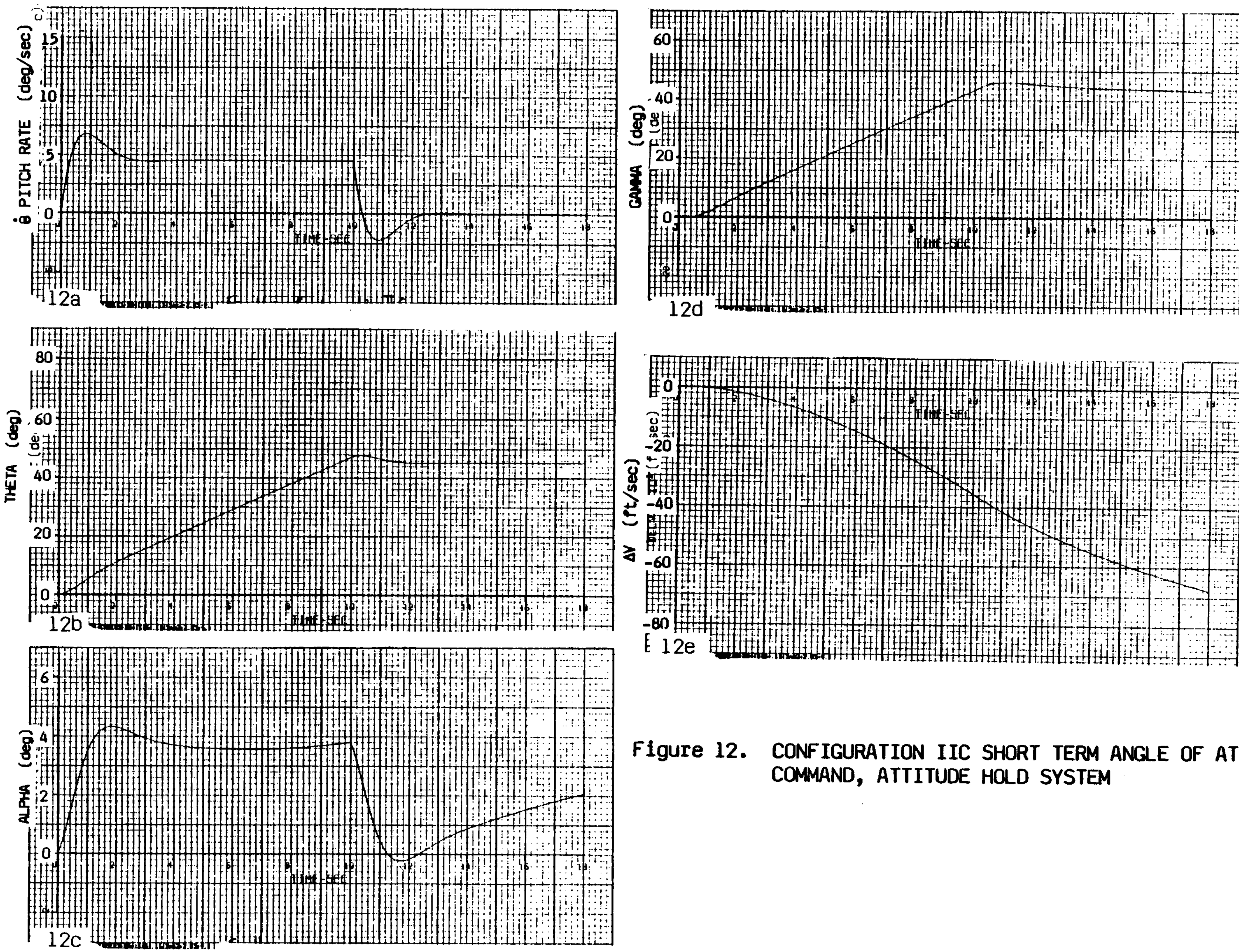


Figure 12. CONFIGURATION IIC SHORT TERM ANGLE OF ATTACK COMMAND, ATTITUDE HOLD SYSTEM

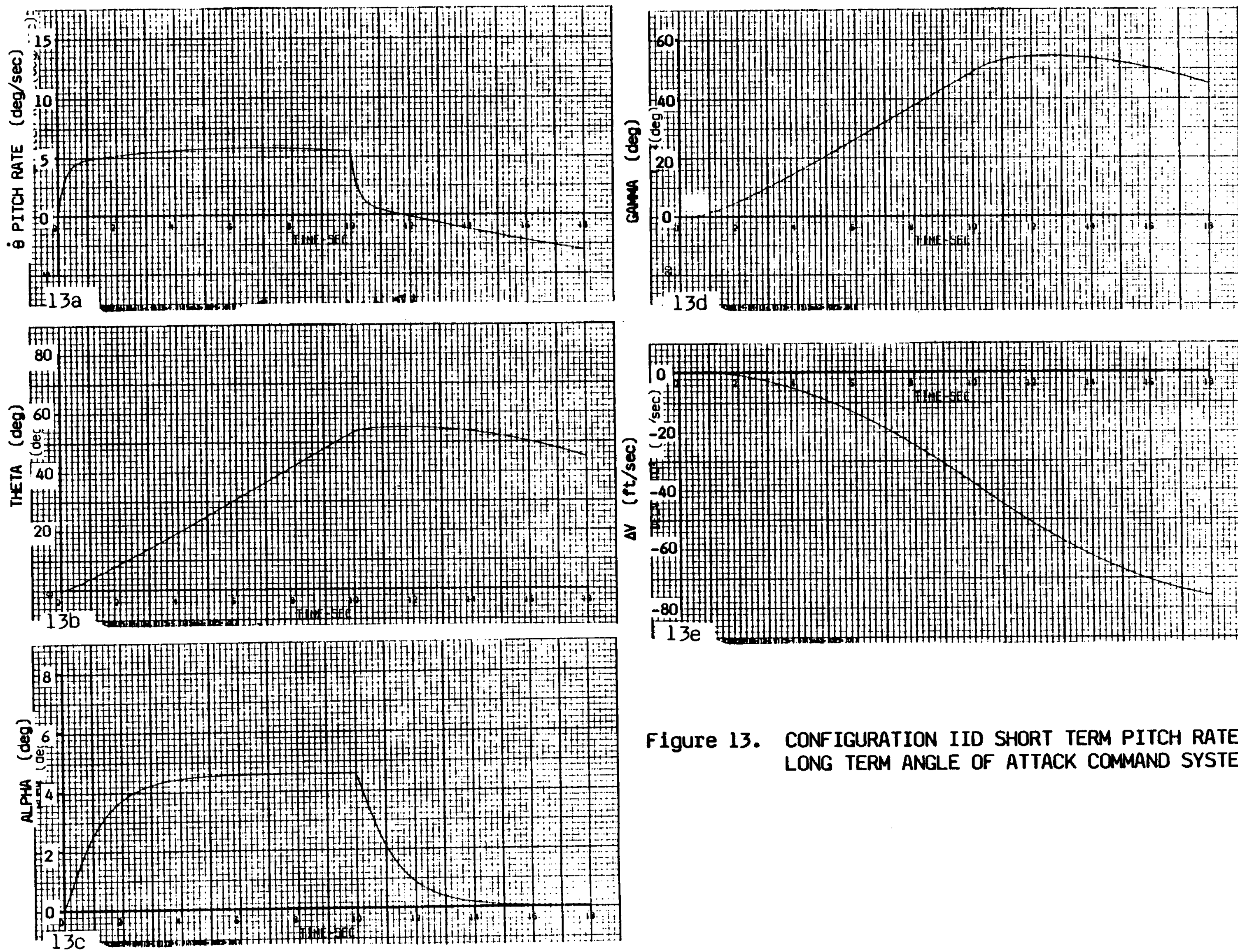


Figure 13. CONFIGURATION IID SHORT TERM PITCH RATE, LONG TERM ANGLE OF ATTACK COMMAND SYSTEM

Section 3
CONTROL SYSTEM DESIGN

3.1 EQUATIONS OF MOTION

The pitch rate and angle of attack command systems for both short period and phugoid behavior and the hybrid pitch rate and angle of attack command system; i.e., pitch rate for short period and angle of attack for phugoid or vice versa, illustrate examples of modal decoupling very clearly and vividly. The pitch rate command system in both short term and long term is characterized by the tranfer function

$$\frac{\dot{\theta}}{\delta_F}(s) = \frac{-20s(s + 1/\tau_{\theta_1})(s + 1/\tau_{\theta_2})}{s(s + 1/\tau_{\theta_1})(s + 1/\tau_{\theta_2})(s + p)} \doteq \frac{20}{s + p} \tag{3-1}$$

while the angle of attack command system is characterized by the transfer function

$$\frac{\alpha}{\delta_F}(s) = \frac{-1.8\left[s^2 + 2\zeta_\alpha\omega_\alpha s + \omega_\alpha^2\right]}{\left[s^2 + 2\zeta_\alpha\omega_\alpha s + \omega_\alpha^2\right]\left[s^2 + 2\zeta_{sp}\omega_{sp} s + \omega_{sp}^2\right]} = \frac{-1.8}{s^2 + 2\zeta_{sp}\omega_{sp} s + \omega_{sp}^2} \tag{3-2}$$

These transfer functions show that no phugoid mode residue appears in the pitch rate response of the pitch rate command system, and no phugoid mode residue appears in the angle of attack response of the angle of attack command system. In effect, the phugoid mode has been decoupled from a particular response variable.

The equations of motion that define the transfer functions of these systems are simply obtained. These equations can then be programmed into the computer of a model following system, such as is available in the TIFS airplane. This would provide flying qualities configurations of aircraft dynamic characteristics independent of a flight control system. This is viewed to be important in a study in which the flying qualities requirements or criteria should be determined independently of a flight control configuration. In Section 3.2.1 of this report, various flight control laws for a particular aircraft are investigated that will yield the dynamic behavior defined either by the transfer functions or the resulting equations of motion. The purpose of doing this is to show that flight control systems can be designed to precisely meet criteria that were independently determined.

The transfer functions for each of the command configurations considered are listed along with the resulting equivalent equations of motion. In each case, a value of $1/\tau_{\theta_2} = 0.9$ and $\omega_p = 0.1$ rad/sec is assumed

A. Pitch Rate Command/Attitude Hold

Transfer Functions:

$$\frac{\dot{\theta}}{\delta}(s) = \frac{-20s(s+0.9)(s+0.1)}{s(s+0.10)(s+0.9)(s+4.4)} \tag{3-3}$$

$$\frac{\alpha}{\delta}(s) = \frac{-1.8(s+10)\left[s^2+0.02s+.01\right]}{s(s+0.10)(s+0.9)(s+4.4)} \tag{3-4}$$

$$\frac{\Delta V}{\delta}(s) = \frac{-25s(s+1)(s-15)}{s(s+0.10)(s+0.9)(s+4.4)} \tag{3-5}$$

The corresponding linear aerodynamics of the equation of motion are:

$$\begin{bmatrix} \dot{q} \\ \dot{\theta} \\ \dot{\alpha} \\ \dot{V} \end{bmatrix} = \begin{bmatrix} -4.4 & 0.0 & 0.0 & 0.0 \\ 1 & 0.0 & 0.0 & 0.0 \\ .497 & -.0676 & -.9023 & -.0008 \\ 1.0315 & -20.5986 & 2.4276 & -.0977 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \theta \\ \alpha \\ \Delta V \end{bmatrix} + \begin{bmatrix} -20.0 \\ 0 \\ -1.80 \\ 0 \end{bmatrix} \delta_e \tag{3-6}$$

B. Angle of Attack Command

The numerators of the transfer function are the same as above. The characteristic polynomial or denominator is

$$D_2(s) = \left[s^2 + 2.8s + 4\right]\left[s^2 + .02s + .01\right] \tag{3-7}$$

The equation of motion becomes:

$$\begin{bmatrix} \dot{q} \\ \dot{\theta} \\ \dot{\alpha} \\ \dot{V} \end{bmatrix} = \begin{bmatrix} -1.5687 & .1090 & -2.7921 & 0.0013 \\ 1.00 & 0.0 & 0.0 & 0.0 \\ .7518 & -.0578 & -1.1536 & -.0006 \\ 1.0315 & -20.5986 & 2.4276 & -.0977 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \theta \\ \alpha \\ \Delta V \end{bmatrix} + \begin{bmatrix} -20.0 \\ 0 \\ -1.80 \\ 0 \end{bmatrix} \delta_e \qquad (3\text{-}8)$$

C. Angle of Attack Short Term Command - Attitude Hold

$$D_3(s) = s(s + 0.01)\left[s^2 + 2.8s + 4.0\right] \qquad (3\text{-}9)$$

Equation of Motion:

$$\begin{bmatrix} \dot{q} \\ \dot{\theta} \\ \dot{\alpha} \\ \dot{V} \end{bmatrix} = \begin{bmatrix} -1.634 & -.2434 & -2.5174 & -.0024 \\ 1.00 & 0.0 & 0.0 & 0.0 \\ .7424 & -.0895 & -1.1289 & -.001 \\ 1.0315 & -20.5986 & 2.4276 & -.0977 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \theta \\ \alpha \\ \Delta V \end{bmatrix} + \begin{bmatrix} -20.0 \\ 0 \\ -1.80 \\ 0 \end{bmatrix} \delta_e \qquad (3\text{-}10)$$

D. Pitch Rate Short Term Command - Long Term Angle of Attack Command

$$D_4(s) = (s + 0.90)(s + 4.4)\left[s^2 + 0.02s + 0.01\right] \qquad (3\text{-}11)$$

Linear aerodynamics of the equation of motion are:

$$\begin{bmatrix} \dot{q} \\ \dot{\theta} \\ \dot{\alpha} \\ \dot{V} \end{bmatrix} = \begin{bmatrix} -4.321 & .3205 & 0.0111 & 0.0037 \\ 1.00 & 0.0 & 0.0 & 0.0 \\ .5041 & -.0387 & -0.9013 & -0.004 \\ 1.0315 & -20.5986 & 2.4276 & -.0977 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \theta \\ \alpha \\ \Delta V \end{bmatrix} + \begin{bmatrix} -20.0 \\ 0 \\ -1.80 \\ 0 \end{bmatrix} \delta_e \qquad (3\text{-}12)$$

These equations represent model aircraft forms that can be programmed directly into the TIFS computer independently of a particular vehicle that the model computer may be intended to represent. Using the method described above, it is then possible to investigate the flying qualities of the different types of command system independently of a particular vehicle and independent of a particular control law.

3.2 FEEDBACK CONTROL LAWS TO SATISFY FLYING QUALITIES REQUIREMENTS

Example feedback control laws for a linearized representation of the AFTI-16 were generated for each of the rate and angle of attack command systems under consideration. The vehicle was assumed to be on a landing approach with a velocity of V=139 knots.

The open loop pitch rate and angle of attack transfer functions were assumed to be

$$\frac{\dot{\theta}}{\delta_e}(s) = \frac{-1.644s(s + .587)(s + .0422)}{D(s)} \tag{3-13}$$

$$\frac{\alpha}{\delta_e}(s) = \frac{-.0717(s + 23.44)(s^2 + .0471s + .037)}{D(s)} \tag{3-14}$$

where D(s) is the open loop characteristic polynomial

$$\begin{aligned} D(s) &= s^4 + 1.06s^3 - 1.116s^2 - .0365s - .0512 \\ &= (s - .724)(s + 1.705)\left[s^2 + .38(204)s + .204^2\right] \end{aligned} \tag{3-15}$$

and shows that the vehicle is statically unstable. From the above transfer functions the zeros of the pitch rate and the angle of attack transfer funtions are $1/\tau_{\theta_1} = .0422$ $\quad 1/\tau_{\theta_2} = 0.587$ $\quad \omega_\alpha = .1924$ $\quad \zeta_\alpha = .122$

Feedback control laws are designed to yield the four closed loop characteristic polynomials defined below:

A. Rate Command/Attitude-Hold

$$\Delta_1(s) = s(s + 1/\tau_{\theta_2})(s + 1/\tau_{\theta_1})(s + p_1) \tag{3-16}$$

The system is designed to have short period dynamics of $\omega_n = 2$. The two short period poles are then located at $s = -1/\tau_{\theta_2}$, $p = \omega_n{}^2\tau_{\theta_2}$. The closed loop characteristic polynomial for the rate command, attitude-hold system becomes

$$\begin{aligned} \Delta_1(s) &= s(s + .587)(s + .0422)(s + 2.348) \\ &= s^4 + 2.9772s^3 + 1.50213s^2 + .05816s \end{aligned} \tag{3-17}$$

B. Angle of Attack Command System

This system is designed such that the closed loop short perod dynamics are $\omega_{sp} = 2$, $\zeta_{sp} = 0.7$. The closed loop characteristic polynomial is

$$
\begin{aligned}
\Delta_2(s) &= \left[s^2 + 2\zeta_{sp}\omega_{sp}s + \omega_{sp}^2\right]\left[s^2 + 2\zeta_\alpha\,\omega_\alpha s + \omega_\alpha^2\right] \\
&= \left[s^2 + 2.8s + 4\right]\left[s^2 + .0471s + .037\right] \\
&= s^4 + 2.8471s^3 + 4.1689s^2 + .2920s + .1480 \qquad (3\text{-}18)
\end{aligned}
$$

C. Short Term Angle of Attack Command/Attitude-Hold

This system is designed to respond during the short term as an angle of attack command system but maintain the attitude-hold feature of the rate command/attitude-hold system. The closed loop characteristic polynomial is defined

$$
\begin{aligned}
\Delta_3(s) &= s(s + 1/\tau_{\theta_1})\left[s^2 + 2\zeta_{sp}\,\omega_{sp}s + \omega_{sp}^2\right] \\
&= s(s + .0422)\left[s^2 + 2.8s + 4\right] \\
&= s^4 + 2.8422s^3 + 4.11816s^2 + 0.1688s \qquad (3\text{-}19)
\end{aligned}
$$

4. Pitch Rate Command Short Term - Angle of Attack Long Term Command

For this system, the closed loop poles are defined

$$
\begin{aligned}
\Delta_4(s) &= (s + 1/\tau_{\theta_2})(s + p_1)\left[s^2 + 2\zeta_\alpha\,\omega_\alpha s + \omega_\alpha^2\right] \\
&= (s + .587)(s + 2.348)\left[s^2 + .0471s + .037\right] \\
&= s^4 + 2.9821s^3 + 1.55356s^2 + .17352s + .0510 \qquad (3\text{-}20)
\end{aligned}
$$

The responses of the closed loop AFTI-16 model are shown in Figure 14 through 17. Four different feedback control laws were designed for each of the four different output response oriented systems shown above; i.e., each of the four different characteristic polynomials defined above.

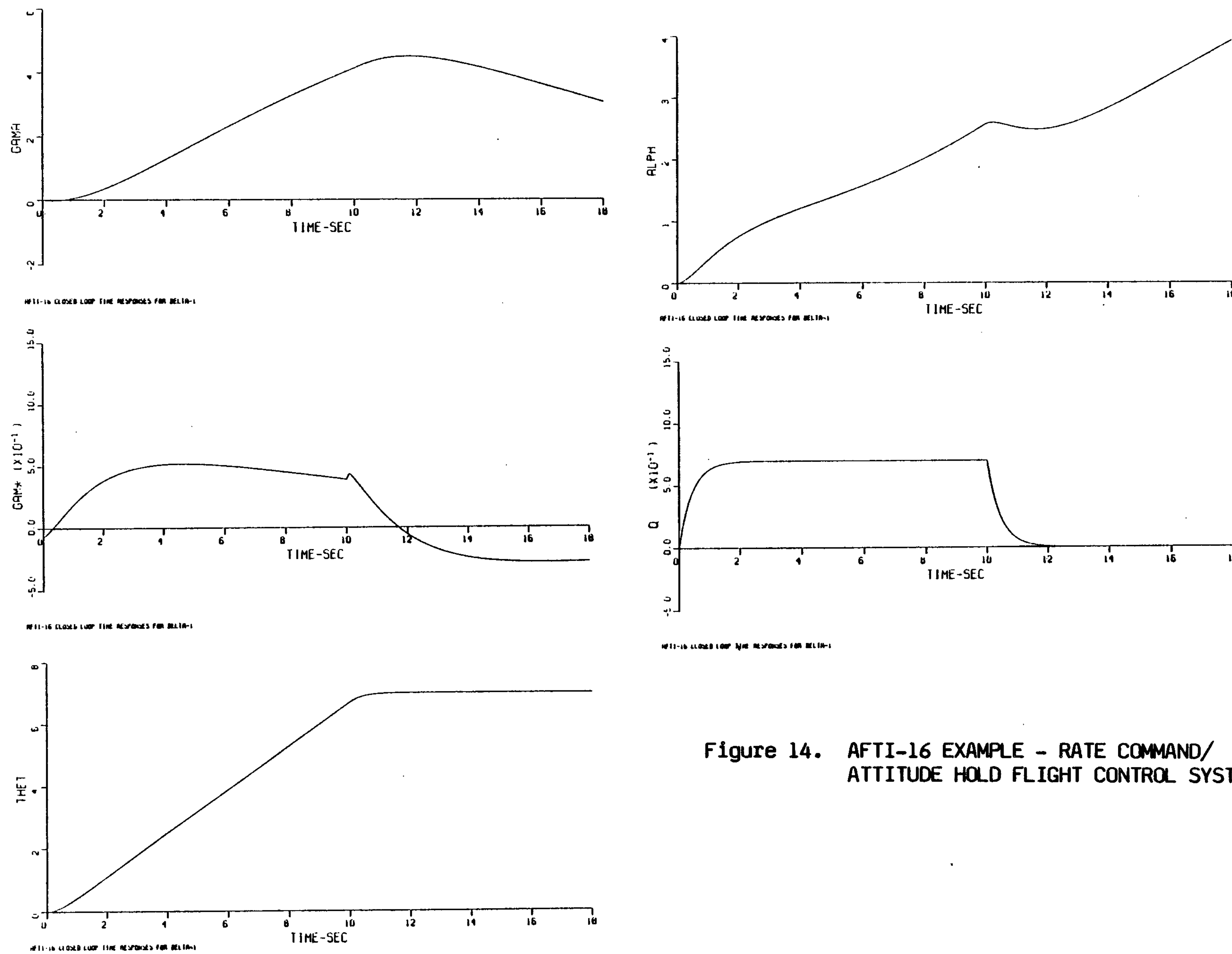


Figure 14. AFTI-16 EXAMPLE - RATE COMMAND/ ATTITUDE HOLD FLIGHT CONTROL SYSTEM

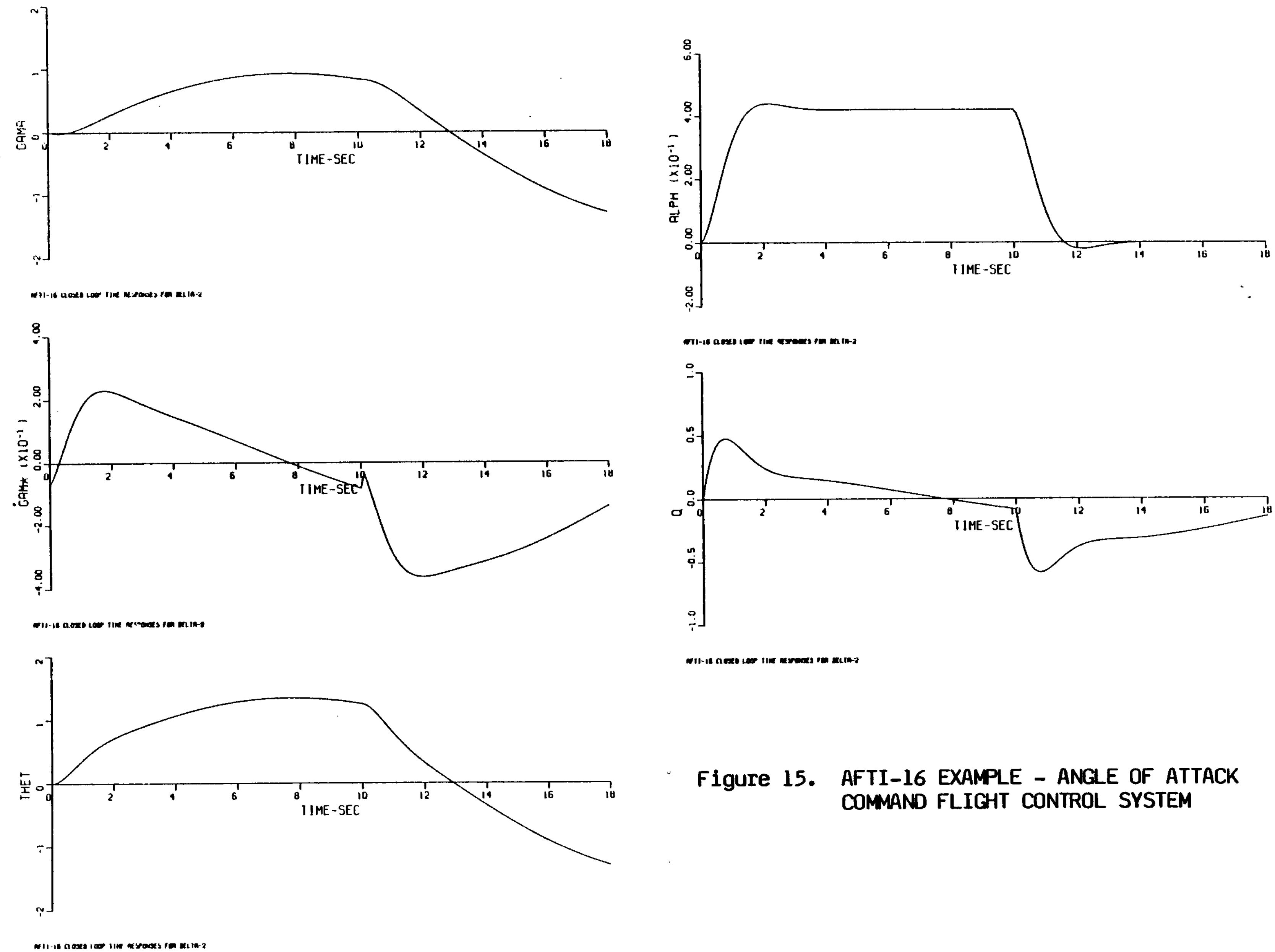


Figure 15. AFTI-16 EXAMPLE - ANGLE OF ATTACK COMMAND FLIGHT CONTROL SYSTEM

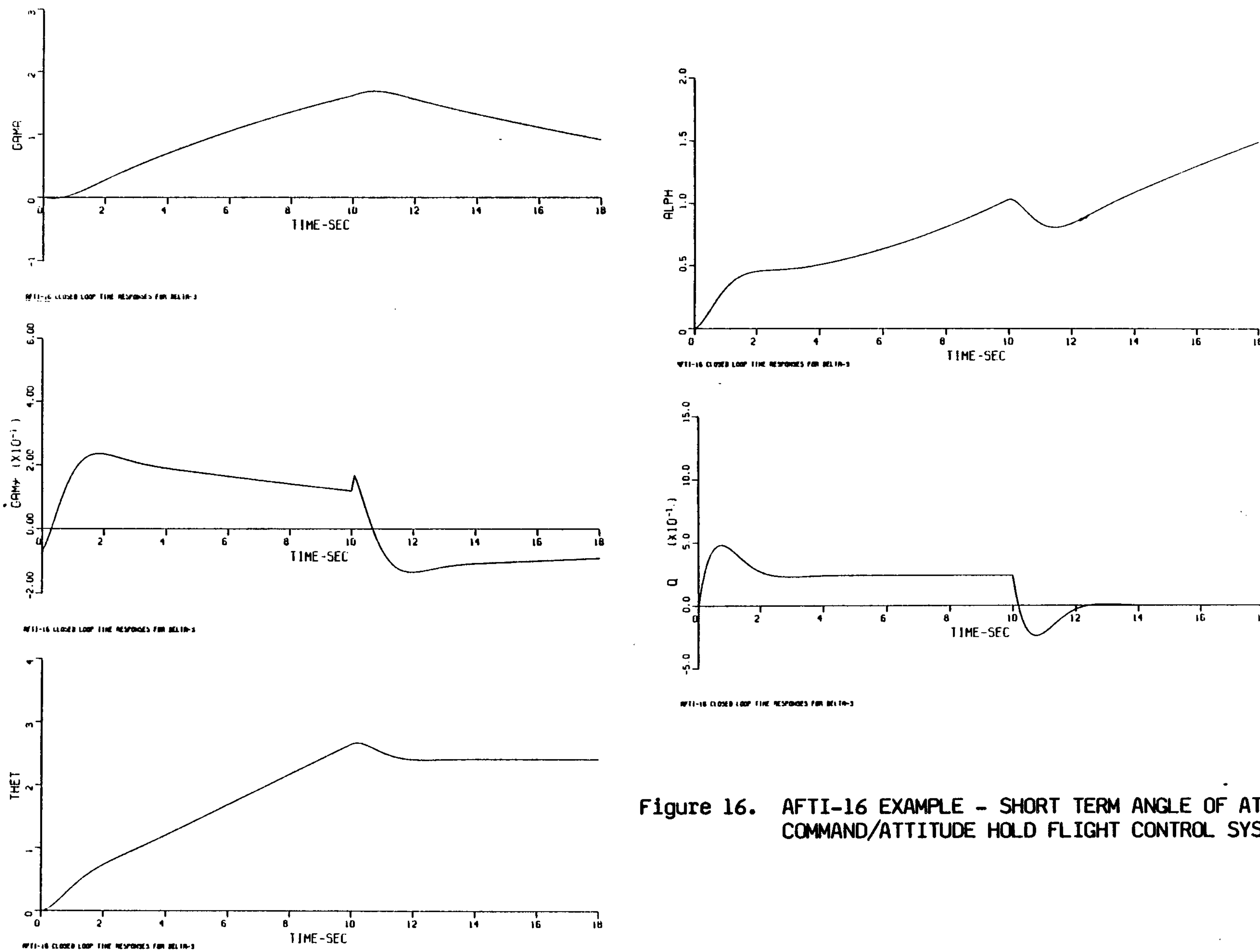


Figure 16. AFTI-16 EXAMPLE - SHORT TERM ANGLE OF ATTACK COMMAND/ATTITUDE HOLD FLIGHT CONTROL SYSTEM

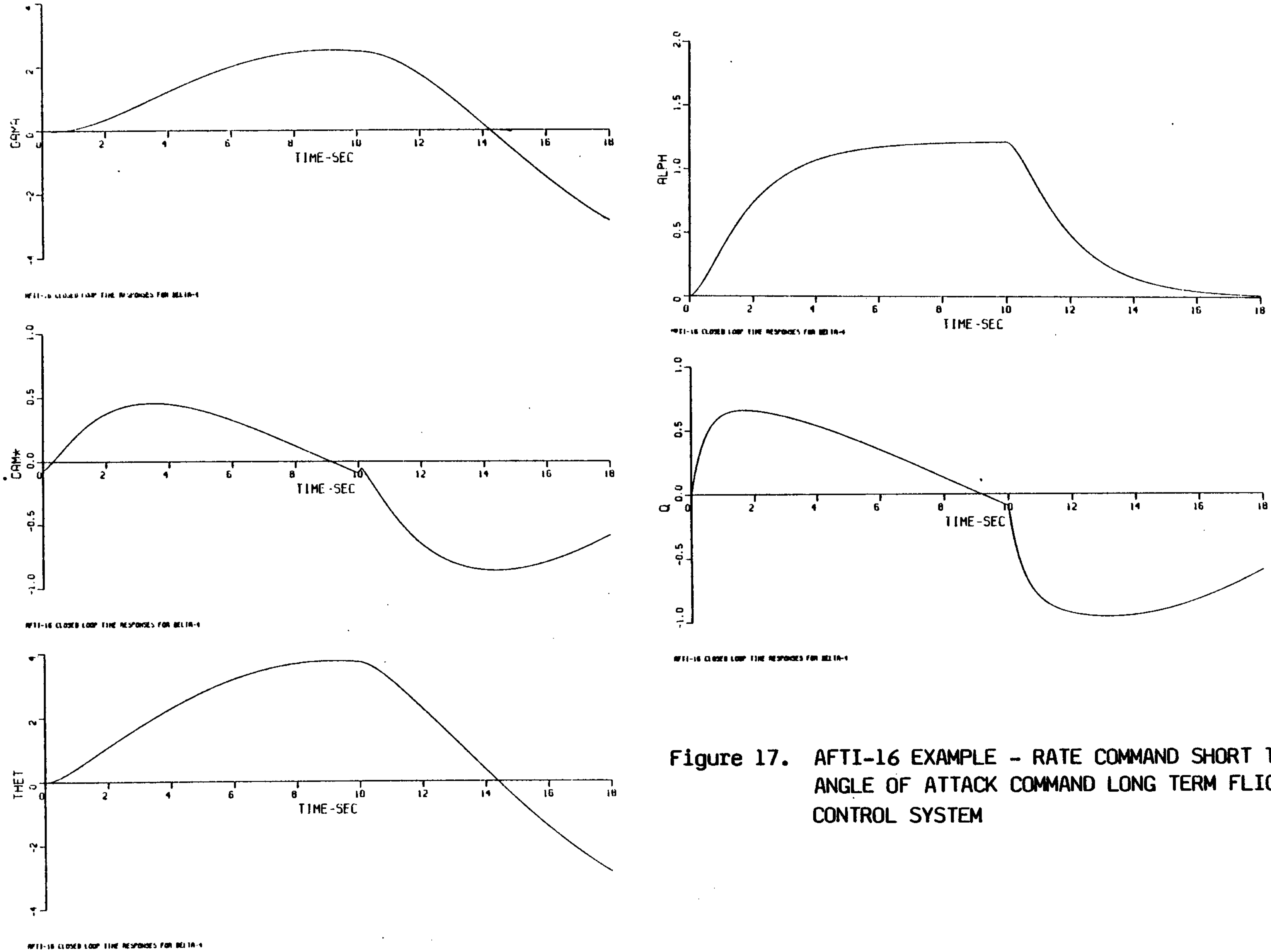


Figure 17. AFTI-16 EXAMPLE - RATE COMMAND SHORT TERM/ ANGLE OF ATTACK COMMAND LONG TERM FLIGHT CONTROL SYSTEM

3.2.1 Control Law Configurations

A variety of control law configurations required to obtain the closed loop dynamics defined in the previous section is possible. Four different control law configurations were selected involving measurements of pitch rate, elevator deflection and in one case, angle of attack. No attempt is made to judge the desirability of any of the control law configurations. Each have their merits and disadvantages but each configuration is represented on an existing airframe. For instance, the feedback of only pitch rate can be representative of an F-16 configuration, while the feedback of pitch rate with elevator measurement is of the same general architecture as the Shuttle flight control law.

The four different control laws involve:

M_1 Angle of attack and pitch rate feedback

M_2 Only pitch rate feedback

M_3 Pitch rate feedback with elevator measurement

M_4 A variation of Control Law M_3, involving different compensation network design

In block diagram form, the four control law configurations are shown in Figures 18 through 21. The four feedback paths of Figures 18 through 21 represent the equivalent of four independent measurements of the system dynamics and all four poles, the short period and the phugoid poles can therefore be "placed," or given any closed loop frequency and damping ratio desired. As shown by the control system conceptual designs of Figures 18 through 21, the poles can be located over a very large range of values - the important problem is to define where they should be located to satisfy flying qualities requirements.

CONFIGURATION M_1

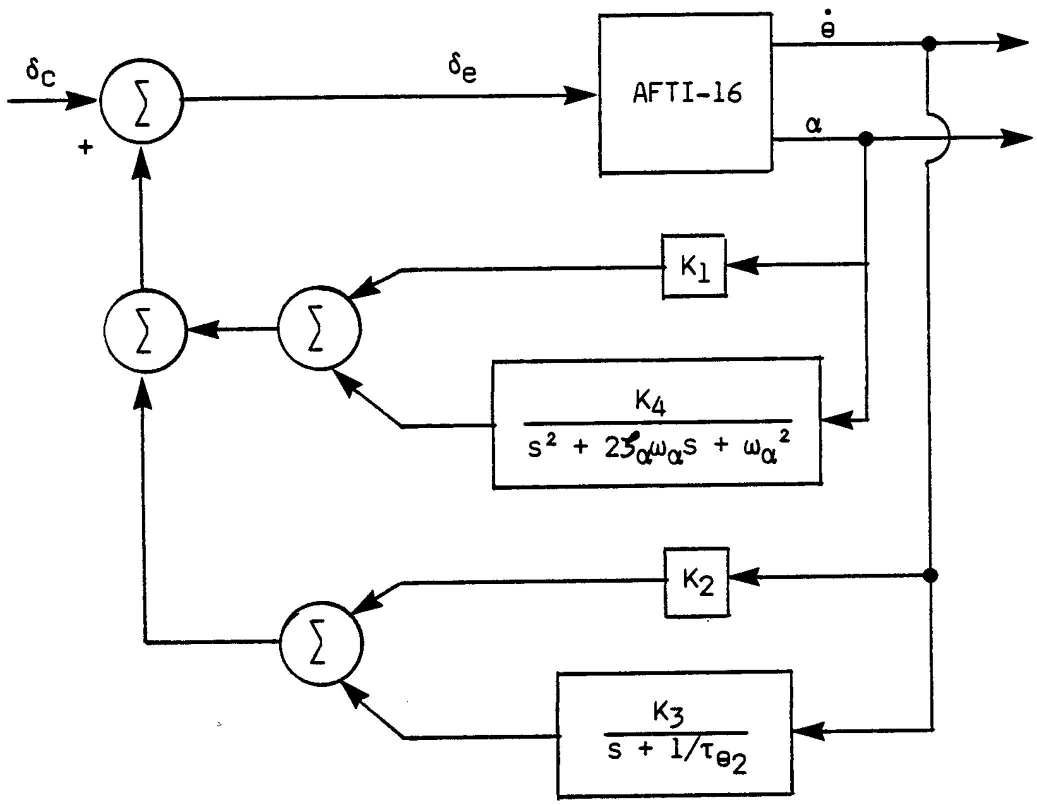


Figure 18. ANGLE OF ATTACK AND PITCH RATE FEEDBACK

CONFIGURATION M_2

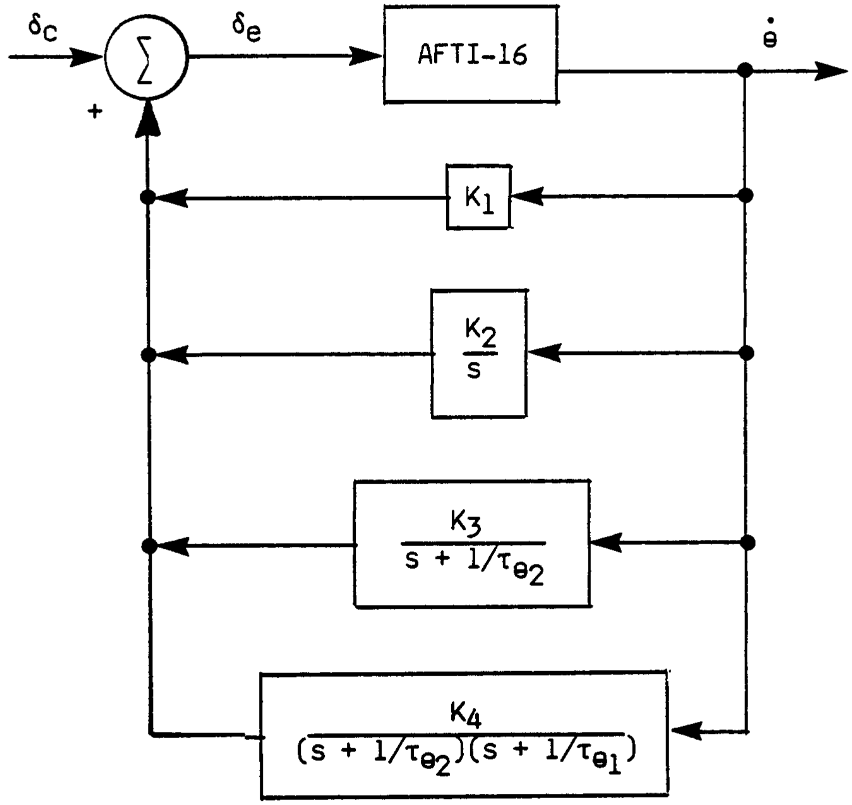


Figure 19. PITCH RATE FEEDBACK

CONFIGURATION M_3

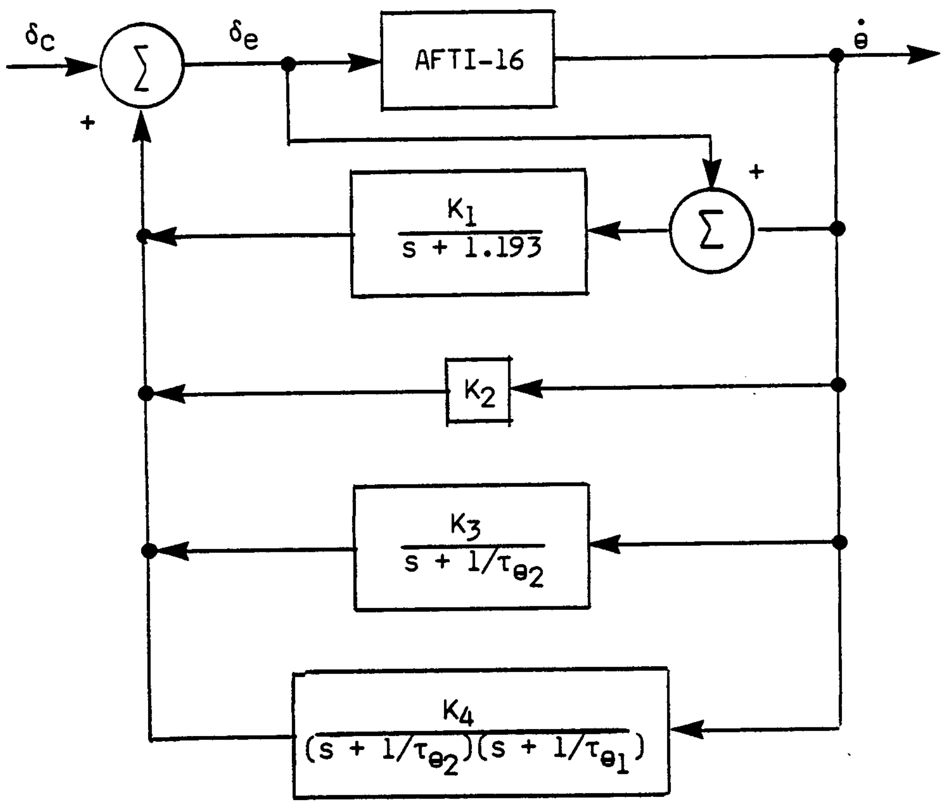


Figure 20. PITCH RATE FEEDBACK WITH ELEVATOR MEASUREMENT

CONFIGURATION M_4

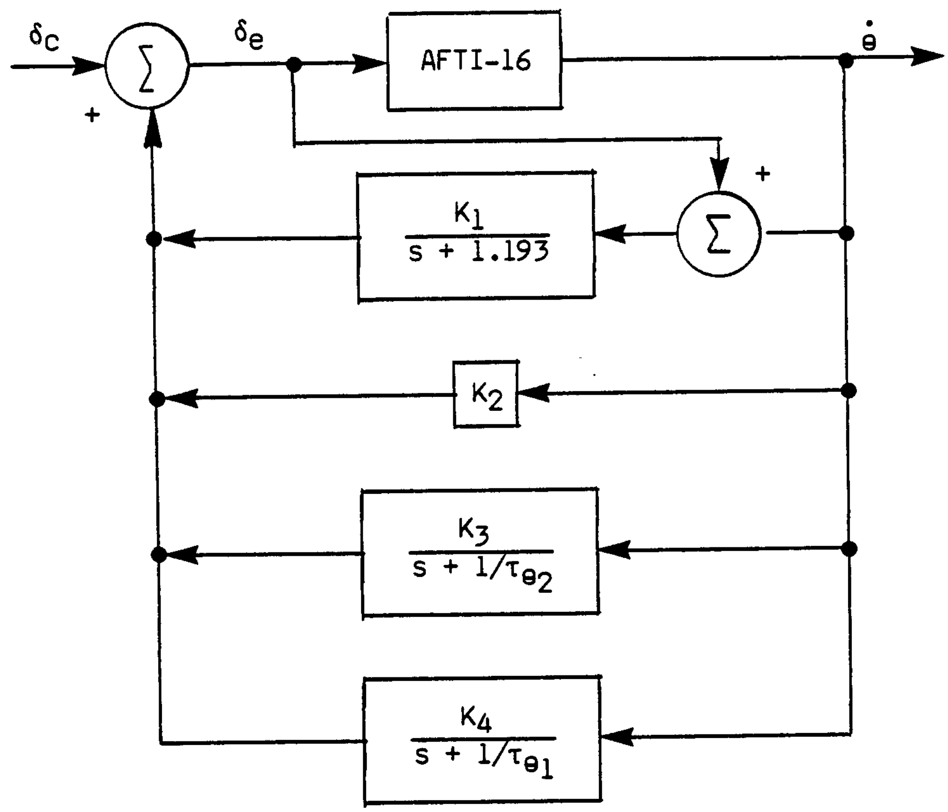


Figure 21. ALTERNATE CONTROL LAW USING PITCH RATE AND ELEVATOR MEASUREMENTS

The feedback control laws employ compensation networks of a special kind of filter called observers. These filters are designed such that a pole of a filter was selected to be equal to a zero of the numerator transfer function of the output quantity fed back. The output of each filter then acts as an independent measurement of the system dynamics. One control law that was not included in the design was the control law that feeds back all the state variables, in this case θ, $\Delta\theta$, ΔV and $\Delta\alpha$. It is not yet considered likely that a full state feedback control law would be implemented by the control system designer.

The control laws are calculated as follows:

The open loop characteristic polynomial is defined as

$$D(s) = s^4 + d_3s^3 + d_2s^2 + d_1s + d_0 \tag{3-21}$$

The desired closed loop characteristic polynomial is defined as

$$\Delta(s) = s^4 + \delta_3 s^3 + \delta_2 s^2 + \delta_1 s + \delta_0 \tag{3-22}$$

The feedback control law is simply calculated by the relationship

$$\delta(t) = -(\Delta - D)M^{-1}x(t) \tag{3-23}$$

where x(t) represents the vector of either the output state fedback or the output of an observer filter. The matrix M is defined from either the coefficient of the numerator polynomial of the transfer function of the sensed output quantity or the coefficients of numerator polynomials of the output of an observer compensation network, all with respect to the elevator input of the airplane. The computational procedure is described in detail in Reference 10.

For example, the control law for the rate command, altitude hold system, defined by Δ_1 and the control law using angle of attack and pitch rate feedback, defined by M_1, is obtained from

$$\delta_e(s) = -(\Delta_1 - D)M_1^{-1} \begin{bmatrix} \alpha(s) \\ \dot{\theta}(s) \\ \dfrac{\dot{\theta}(s)}{s + 1/\tau_{\theta_2}} \\ \dfrac{\alpha(s)}{s^2 + 2\zeta_\alpha \omega_\alpha s + \omega_\alpha^2} \end{bmatrix} \tag{3-24}$$

$$= -[.0512, .09466, 2.61813, 1.9172] \begin{bmatrix} -.0622 & -.0852 & -1.683 & -.0717 \\ 0 & -.0852 & -1.032 & -1.644 \\ 0 & -.0694 & -1.644 & 0 \\ -1.6906 & -.0717 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} \alpha(s) \\ \dot{\theta}(s) \\ \dfrac{\dot{\theta}(s)}{s + 1/\tau_{\theta_2}} \\ \dfrac{\alpha(s)}{s^2+2\zeta_\alpha \omega_\alpha s+\omega_\alpha^2} \end{bmatrix}$$

$$\delta_e(s) = -1.2619\ \alpha(s) + 1.2212\ \dot{\theta}(s) + \frac{2.117}{s+.587}\ \dot{\theta}(s) + \frac{.0767}{s^2+.0471s+.037}\ \alpha(s) \tag{3-25}$$

The feedback gains required are quite reasonable in magnitude with units of deg/deg/sec or deg/deg.

The control laws for each of the four different closed loop characteristic polynomials are defined in Tables 2 through 5 below.

Table 2

CONTROL LAWS FOR $\Delta_1(s)$

Configuration	Gains			
	K_1	K_2	K_3	K_4
M_1	-1.2619	1.2212	2.117	1.9252
M_2	1.1662	.1258	-1.1906	1.9252
M_3	1.1689	1.8772	-.2085	-.6409
M_4	1.1689	1.8772	-.899	-.04966

Table 3

CONTROL LAWS FOR $\Delta_2(s)$

Configuration	Gains			
	K_1	K_2	K_3	K_4
M_1	8.5966	.71212	-6.0329	-.19845
M_2	1.0870	.4894	-3.182	5.225
M_3	4.5479	3.8534	.63866	-4.7588
M_4	4.5479	3.8534	-4.4889	3.6876

Table 4

CONTROL LAWS FOR $\Delta_3(s)$

Configuration	Gains			
	K_1	K_2	K_3	K_4
M_1	-1.2646	1.1392	3.7633	.0768
M_2	1.0841	.1258	.4518	1.9257
M_3	1.1689	1.795	1.434	-.6404
M_4	1.1689	1.795	.7438	.04966

Table 5
CONTROL LAWS FOR $\Delta_4(s)$

Configuration	Gains			
	K_1	K_2	K_3	K_4
M_1	8.276	.8082	-7.356	-.244
M_2	1.169	.2511	-1.393	2.032
M_3	2.333	2.588	.5667	-3.089
M_4	2.333	2.588	-2.763	2.394

The Control law tables indicate that all of the control laws are mechanizable for each of the systems involving pitch rate or angle of attack command. The sixteen control laws shown are only examples of what can be obtained. A near-infinity of other combinations are also possible and only a few have been illustrated above, so therefore the real purpose of these calculations is to demonstrate that flying qualities criteria requirements need not be closely connected to the sensors used for feedback or the particular system architecture.

3.3 PREFILTER DESIGN TECHNIQUES

The flight control system designs of Figures 18 through 21 show by example that rate command or path command or a hybrid system involving rate or path in the short term and path or rate in the long term can be obtained using a variety of sensors for system configuration purposes. In effect, it is not necessary, although usually desirable to feed back a "controlled" quantity. As the previous examples show, it is entirely possible to separately specify flying qualities criteria for one response variable and specify a feedback control configuration involving an entirely different quantity with compensation networks designed as observers.

system designer to satisfy particular criteria is not limited to feedback quantities. The dynamics of a particular system in response to a pilot command can, of course, be shaped through the use of prefilters, even to the point of altering a rate command system using a pitch rate gyro and forward loop integrator such that the resulting system behaves as a path command or "conventional aircraft" system in response to a pilot input. The technique is not new, the inclusion of a prefilter is a standard flight control system design method, but has not been previously used to specifically meet criteria formulated independently.

As previously shown, the definition of a rate command or an angle of attack (flight path) command system has to do with the location of the closed loop poles with respect to the numerator zeros of the transfer functions. It is a relatively straightforward design technique to use a prefilter that tends to relocate response poles by pole-zero cancellation methods. A prefilter containing a zero equal to a closed loop pole cancels that pole in the response to command inputs. A prefilter pole then is used to shape the response to a command input by the pilot.

As an example, consider the system shown in Figure 22 below. The configuration utilizes a proportional plus integral network in the feedforward path and pitch rate feedback

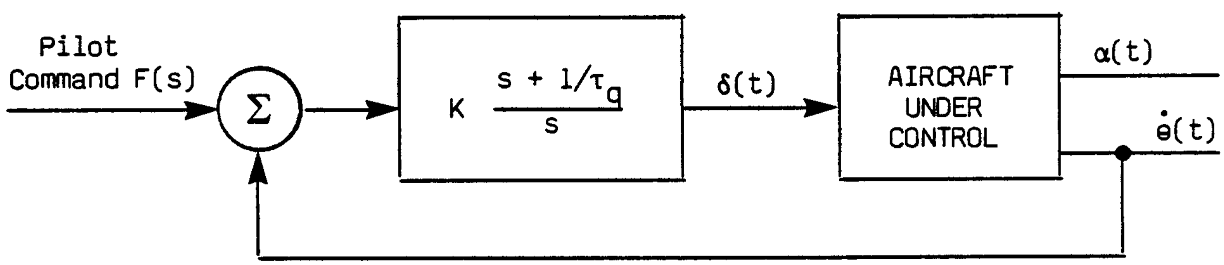


Figure 22. BASIC RATE COMMAND/ATTITUDE HOLD SYSTEM

The closed loop transfer function for pitch rate is:

$$\frac{\dot{\theta}}{F}(s) = K_1 \frac{s(s + 1/\tau_{\theta_2})(s + 1/\tau_{\theta_1})(s + \tau_q)}{s\,(s + T_1)(s + T_2)\left[s^2 + 2\zeta_q\,\omega_q\,s + \omega_q^{\,2}\right]} \qquad (3\text{-}26)$$

where $1/T_1 \doteq 1/\tau_{\theta_1}$ and $1/T_2 \doteq 1/\tau_{\theta_2}$. Therefore, the pitch rate transfer function can be approximated by

$$\frac{\dot{\theta}}{F}(s) \doteq K_1 \frac{(s + 1/\tau_q)}{[s^2 + 2\zeta_q \omega_q s + \omega_q^2]} \tag{3-27}$$

The angle of attack transfer function is given by

$$\frac{\alpha}{F}(s) = \frac{K_2(s + 1/\tau_q)(s + 1/\tau_\alpha)[s^2 + 2\zeta_\alpha \omega_\alpha s + \omega_\alpha^2]}{s(s + 1/T_1)(s + 1/T_2)[s^2 + 2\zeta_q \omega_q s + \omega_q^2]} \tag{3-28}$$

and cannot be reduced by approximation in a manner similar to the way the pitch rate transfer was simplified. A prefilter designed to pole-zero cancel the poles at $-1/T_1$, $-1/T_2$ and at the origin and cancel the zeros defined by $-1/\tau_q$ and $[s^2 + 2\zeta_\alpha \omega_\alpha s + \omega_\alpha^2]$ will yield a simplified angle of attack transfer function and change the system from a rate command to an angle of attack command system. A block diagram containing the required prefilter is shown in Figure 23 below.

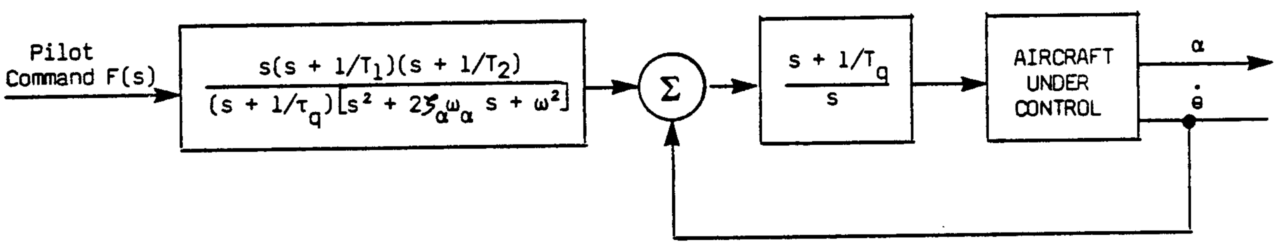


Figure 23. ANGLE OF ATTACK COMMAND SYSTEM USING PITCH RATE FEEDBACK AND PREFILTER

The angle of attack transfer function to a pilot command input is therefore given by

$$\frac{\alpha}{F}(s) = \frac{s(s + 1/T_1)(s + 1/T_2)}{(s + 1/\tau_q)[s^2 + 2\zeta_\alpha\omega_\alpha s + \omega_\alpha^2]} \cdot \frac{K_2(s + 1/\tau_q)(s + 1/\tau_\alpha)[s^2 + 2\zeta_\alpha\omega_\alpha s + \omega_\alpha^2]}{s(s + 1/T_1)(s + 1/T_2)[s^2 + 2\zeta_q\omega_q s + \omega_q^2]}$$

$$\doteq \frac{K_2(s + 1/\tau_\alpha)}{[s^2 + 2\zeta_q \omega_q s + \omega_q^2]} \tag{3-29}$$

The prefilter has been used to change a rate command, attitude-hold system using pitch rate feedback to one that commands angle of attack or path. It should be emphasized that the pole-zero cancellations need not be exact. An inexact cancellation will produce relatively small residues in the prefilter poles.

There are several major considerations that need be noted in a prefilter or feedback design:

1. The prefilter affects only the response to a command input. Response to disturbances that enter directly into the vehicle aerodynamics, such as turbulence, will not be affected by the prefilter. For the example shown in Figure 23 above, the vehicle will respond to command inputs as a path command system, or a "conventional" aircraft, but will respond to turbulence as a rate command, attitude hold system.

2. A prefilter can be used to shape the command input to satisfy flying qualities requirements almost independently of how poor the flying qualities of the closed loop part of the system might be. For instance, it may be to use limited feedback gains for vehicles that are highly flexible. The objective could be to provide for basic rigid body stability of statically unstable vehicles, then provide satisfactory and acceptable flying qualities using a prefilter. In this way, the possibility of an aeroservoelastic instability may be minimized and aeroelastic compensation can be kept simple.

3. The prefilter/feedback design methods described above are known as model following techniques. A prefilter designed as hown in Figure 23 above is referred to as explicit model following, while the feedback method of obtaining exactly the closed loop dynamics desired is known as implicit model following. These methods are methods of superaugmentation in the sense that the desired dynamic and static behavior in response to a pilot command are defined apriori, and the design technique is exact and closed form. No guess and test methods are required. In fact, the dynamic behavior of one degree-of-freedom per independent controller can be specified beforehand, and the systems can be mechanized either using feed-

back (implicit model following) or as a prefilter (explicit model following). Either method is feasible as a flight control system. Explicit model following is exemplified by the USAF/AFWAL Total In-Flight Simulator (TIFS) that has been successfully flying for more than ten years, while the NT-33A, which has been flying for more than twenty-five years, is an excellent example of the implicit model following design approach.

4. The prefilter/feedback versatility of a flight control system design offers a method of providing for a spectrum of response characteristics of an airplane without altering the feedback configuration of the vehicle. A typical design concept for a task-oriented flight control system would contain feedback for the purpose of satisfactory and acceptable flying qualities for most of the general, up and away flight tasks, while a series of prefilters can be used to optimize the vehicle response for particular tasks. For instance, general maneuvering would require an angle of attack or path command system, while the specialized task of firing a laser weapon, for instance, would likely require precise attitude command. A prefilter could be used to temporarily change the system into an attitude command system.

3.4 DECOUPLING

3.4.1 General Comments

It has been felt and often stated that an aircraft decoupled in response to pilot command inputs would prove to have superior flying qualities. The limited number of flight experiments to date have demonstrated several characteristics of decoupling that had been previously postulated:

1. Not all decoupling is useful or desirable. The flight experiments of lateral-directional decoupling demonstrated using the NT-33A (Reference 6) revealed that the pilots felt a flat turn capability to be very useful, contributing significantly to flight precision. However, they found no particular advantage to the side-step or decoupled lateral velocity capability. This side-step mode was not investigated extensively, however, so the results must be considered preliminary in nature.

2. Very precise control is possible for some configurations that employ decoupling. The results of the experiments described in Reference 7 indicated that the best flying qualities attainable for the decelerating V/STOL landing task were obtained for a system that decoupled flight path and velocity. A throttle controller that produced velocity changes with very little perturbation in flight path was rated very highly by the pilots. A stick that produced changes only in flight path with very little changes, either long term or short term, in velocity was also rated very highly by the pilots during the experimental flight program.

3. The accuracy of the decoupling or "purity" is extremely important. Flight experiments in which the "purity" of the decoupling was insufficient resulted in relatively little improvement in the flying qualities of the vehicle.

Because a separate cockpit controller must be provided for the pilot for each decoupled response variable, complete and independent decoupling of all six degrees of freedom of motion would require six separate cockpit controllers. In addition, a control system designed to decouple degrees of freedom of vehicle motion is complex, requiring either high feedback gains or precise knowledge of the vehicle stability and control parameters to achieve a high level of "purity". The major thrust of an investigation should then define the following:

1. Which response variables, if decoupled, would result in exceptional improvement in either flying qualities or task performance.

2. How "pure" should the decoupling be.

The results of the type of investigation described above will have a substantial effect on the complexity of any flight control system design. Only until the questions asked above have been answered with confidence can the proper trade-off between complexity and improved performance be made.

3.4.2 Types of Decoupling

It is generally accepted that the term "decoupling" as applied to a linear system can have four distinct and different meanings. These four different meanings of decoupling are listed below in order to clarify and categorize the different way the word is commonly used. The definitions below are given in the order of complexity required of a flight control system design to achieve the decoupling.

1. Force and Moment or Effector Decoupling

   This type of decoupling is meant to describe a flight control system that has been designed such that a pilot command input produces either a pure force along a vehicle axis system or a pure moment about a vehicle axis. For instance, an elevator and a direct lift device can be interconnected such that a pilot stick input produces only a pitching moment about the Y axis of the airplane or a vertical force along the Z axis of the vehicle.

2. Static or Steady State Decoupling

   Static or steady state decoupling implies interconnections among the control effectors such that only one response variable will take on a new steady state value in response to a pilot step command input. For instance, the controllers could be interconnected such that a pilot stick step command input would result in a steady state change in vertical velocity with no steady state change in attitude or forward velocity component.

3. Modal Decoupling

   Modal decoupling implies the design of a control system such that a response variable of the aircraft contains no residue in one of the natural modes of response of the vehicle. For instance, a control system configuration can be specified that will result in no long term or phugoid mode motion in the angle of attack of the vehicle in response to a pilot step command input.

4. Dynamic Decoupling

Dynamic decoupling implies a control system design such that only one state responds, both statically and dynamically, to a pilot command input. For example, a system can be designed such that a pilot stick command input produces a change only in pitch attitude, with no dynamic or static perturbation in either angle of attack or velocity.

3.4.3 Design Techniques for Decoupling

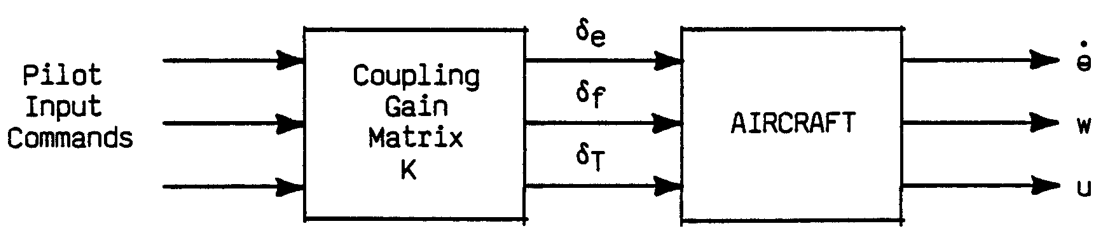
Each of the four types of decoupling defined above can best be demonstrated using state space or matrix-vector methods. The small perturbation equations of vehicle motion are expressed in the familiar matrix-vector format:

$$\dot{x} = Fx + G\,\delta$$

where x is the state vector, usually specified as $x^T = (q, \theta, w, u)$ in the longitudinal-vertical motions in a body axis system centered through the vehicle center of gravity. The vector δ defines the control inputs to the vehicle, typically $\delta^T = (\delta_e, \delta_f, \delta_T)$ where δ_e represents an elevator deflection, δ_f a direct lift flap deflection and δ_T a thrust change. The matrix F is a matrix of aerodynamic and gravitational dimensional stability derivatives, while G defines the control effectiveness term; i.e., the control derivatives.

3.4.3.1 Force-Moment Decoupling

Force and moment decoupling is accomplished by cross coupling among the available effectors, such as elevator, throttle and direct lift flap such that the control effectiveness matrix appears diagonal to a pilot command input. The system is shown in the sketch below



Mathematically, the interconnections are defined by the matrix K, where

$$GK = A \qquad (3\text{-}30)$$

and where A is a diagonal matrix of the force and moment derivative matrix

$$A = \begin{bmatrix} M'_{\delta_e} & 0 & 0 \\ 0 & Z'_{\delta_f} & 0 \\ 0 & 0 & X'_{\delta_T} \end{bmatrix} \tag{3-31}$$

The interconnecting matrix K that produces a force and moment decoupling is then simply defined by

$$K = G^{-1}A \tag{3-32}$$

In terms of the small perturbation motion equations, the system becomes

$$\begin{aligned} \dot{x} &= Fx + G\,\delta \\ \delta &= K\,\delta_c \\ \dot{x} &= Fx + G\,K\,\delta_c \\ &= Fx + G\,G^{-1}A\,\delta_c \\ &= Fx + A\,\delta_c \end{aligned} \tag{3-33}$$

and the sytem has been force and moment decoupled.

This type of decoupling has very definite potential. For instance, the lift due to an elevator deflection produces the non-minimum phase response in normal acceleration and the lag in the flight path response. The type of force and moment decoupling described about will have the effect of eliminating the Z_{δ_e} effect to pilot commands, therefore reducing the lag in the flight path response of the vehicle, with subsequent improvement in the precision of flight.

3.4.3.2 Steady State or Static Decoupling

Using the state space equations, the solution for the interconnecting gain matrix K that will produce a steady state response in only one state variable is described as

$$\dot{x} = Fx + G\,\delta$$
$$\delta = K\,\delta_C$$

or

$$\dot{x} = Fx + G\,K\,\delta_C \tag{3-34}$$

A steady state gain value $X_{i_{ss}}$ is chosen and all other states and state rates are set to zero to yield

$$0 = F\,X_{ss} + G\,K_i\,\delta_{c_i} \tag{3-35}$$

where K_i is a column of the matrix K to be determined. From Equation 3-35,

$$0 = F\,X_{ss}/\delta_{i_c}(s) + G\,K_1$$

or

$$K_i = -G^{-1}F\,X_{i_{ss}}/u_{i_c}(s) \tag{3-36}$$

Example

Consider the simplified two degree of freedom equations of longitudinal vertical motion

$$\dot{q} = M_q q + M_w w + M_{\delta_e}\delta_e + M_{\delta_z}\delta_z \tag{3-37}$$

$$\dot{w} = Uq + Z_w w + Z_{\delta_e}\delta_e + Z_{\delta_z}\delta_z \tag{3-38}$$

Assume that it is desirable to produce a steady state pitch rate but zero steady state vertical velocity to a pilot stick pitch command input. Then $\dot{q} = \dot{w} = w = 0$. Equation 3-38 becomes

$$\begin{matrix} 0 = \\ 0 = \end{matrix} \begin{bmatrix} M_q q_{ss} \\ U_o q_{ss} \end{bmatrix} + \begin{bmatrix} M_{\delta_e} & M_{\delta_z} \\ Z_{\delta_e} & Z_{\delta_z} \end{bmatrix} \begin{bmatrix} K_{11} \\ K_{21} \end{bmatrix} \delta_{e_c} \tag{3-39}$$

Then, from Equation 3-39 the solution for the intercoupling gain K_{11} and K_{21} becomes

$$\begin{bmatrix} K_{11} \\ K_{21} \end{bmatrix} = -\begin{bmatrix} M_{\delta_e} & M_{\delta_z} \\ Z_{\delta_e} & Z_{\delta_z} \end{bmatrix}^{-1} \begin{bmatrix} M_q \\ U_o \end{bmatrix} q_{ss}/\delta_{e_c}$$

or

$$\begin{bmatrix} K_{11} \\ K_{21} \end{bmatrix} = -\frac{1}{M_{\delta_e}Z_{\delta_z} - M_{\delta_z}Z_{\delta_e}} \begin{bmatrix} -\,Z_{\delta_z}M_q - M_{\delta_z}U_o \\ -\,Z_{\delta_e}M_q - M_{\delta_e}U_o \end{bmatrix} q_{ss}/\delta_{e_c} \tag{3-40}$$

The interconnecting gains K_{11} and K_{21} defined above will produce a steady pitch rate q_{ss}/δ_{e_c} value per unit stick command deflection. The vertical velocity in the steady state will be zero, although dynamic perturbation transients will, in general, occur.

3.4.3.3 Modal Decoupling

Modal decoupling is a form of semi-dynamic decoupling in which a certain mode of the aircraft response, such as phugoid or short period mode does not appear in one of the responses of the vehicle. For instance, modal decoupling has been achieved when no phugoid motions appear in the angle of attack response to a pilot command input. This method is the method used to design the rate and angle of attack command systems described in Section 2 of this report and additional elaboration will not be included at this point.

3.4.3.4 Complete Dynamic Decoupling

Complete dynamic decouping involves feedback and/or feedforward compensation designed in such a way that the aerodynamic coupling terms in the equations of motion are eliminated. This can be done most efficiently and effectively using model following methods. Although the model following design method is the most effective because, as shown below, the sensitivity to knowledge of the stability and control derivatives is minimized, a design can be obtained directly as described in Reference 6.

3.4.4 Implicit Model Following

Model following design techniques involve the apriori specification of the desired dynamic behavior of an aircraft. A control law is then defined that will force the vehicle under control to respond as the model responds. In general, the vector-matrix representation of a model, subscript m, can be defined as

$$\dot{x}_m(t) = F_m\, x_m(t) + G_m\, \delta_m(t) \tag{3-41}$$

while the vehicle for which the decoupling system is to be designed has a parallel mathematical model description.

$$\dot{x}(t) = F\, x(t) + G\, \delta(t) \tag{3-42}$$

If a control law can be defined that will force the vehicle under control to respond as the model, then it is a relatively simple matter to apriori define a set of decoupled equations of motion and design the control law to force the vehicle to respond as the model.

The requirements of model following are that the vehicle states and state rates be equal to that of the model; i.e., $x_m(t) = x(t)$ and $\dot{x}_m(t) = \dot{x}(t)$. If $\dot{x}_m(t) = \dot{x}(t)$ and $x_m(t) = x(t)$ then Equation 3-41 and 3-42 can be equated in both $\dot{x}$ and x to yield

$$F\ x(t) + G\ \delta(t) = F_m\ x(t) + G_m\ \delta_m(t) \tag{3-43}$$

It is then straight forward to solve for the control input u(t) that satisfies the above equation. The solution is

$$G\ \delta(t) = (F_m - F)\ x(t) + G_m\ \delta_m(t)$$

or

$$\begin{aligned} \delta(t) &= G^{-1}[(F_m - F)\ x(t) + G_m\ \delta_m(t)] \\ &= (G^T\ G)^{-1}G^T\ [(F_m - F)\ x(t) + G_m\ \delta_m(t)] \\ &= K_1\ x(t) + K_2\ \delta_m(t) \end{aligned} \tag{3-44}$$

which indicates that the control law is a feedback and feedforward control law involving, in the general case, all the state variables of the vehicle under control.

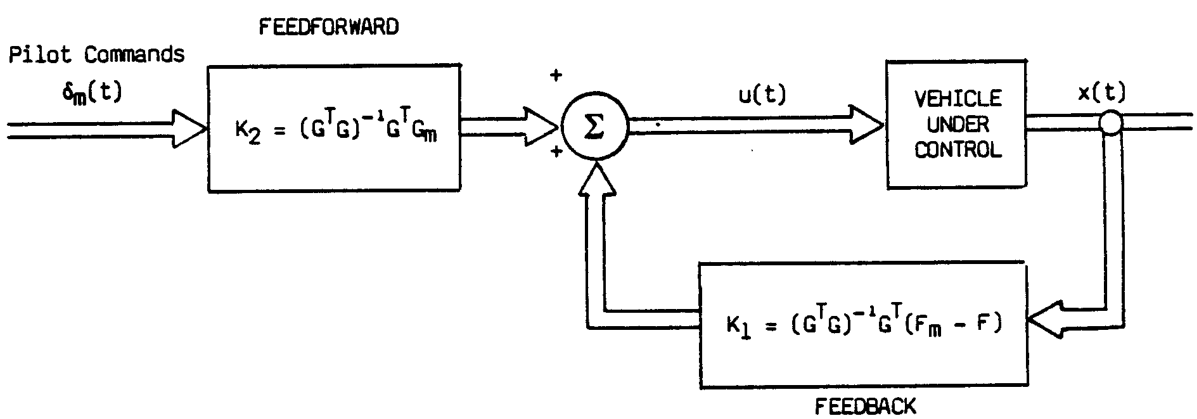


Figure 24. BLOCK DIAGRAM OF IMPLICIT MODEL FOLLOWING SYSTEM

Although the method described is very general in that it is assumed that an independent controller or force and moment producing device is available for each degree of freedom of motion represented, the system can be defined for partial decoupling or model following when fewer controllers are available than degrees of freedom of motion. This is done simply by defining some of the stability and control derivatives of the model to be equal to those of the aircraft under control in such a way that feedback is defined only to the available controllers. The USAF NT-33A control system is designed as an implicit model following system.

3.4.5 Explicit Model Following

The method of implicit model following defined above has the advantage of simplicity in that all the feedforward and feedback elements are gains, but it is sensitive or non-robust in that relatively precise knowledge of the stability and control derivatives of the vehicle are required and must be gain scheduled as a function of flight condition.

Explicit or prefilter model following, as shown below, has the advantage of being less sensitive to knowledge of the stability and control parameters of the airplane but has the disadvantage of requiring a more complex control law involving a dynamic model. Explicit model following is also obtained from Equations 3-41 and 3-42. Assuming $\dot{x}_m(t) = \dot{x}(t)$, $x_m(t) = x(t)$, then $\dot{x}_m(t)$ and $x_m(t)$ can be substituted into Equation 3-42 to obtain

$$\dot{x}_m(t) = F\, x_m(t) + G\, \delta(t)$$

The control motion $\delta(t)$ that will force the vehicle to respond as the model responds is then obtained by solving Equation 3-45 above for $\delta(t)$

$$G\, \delta(t) = \dot{x}_m(t) - F\, x_m(t)$$

or

$$\delta(t) = (G^T G)^{-1} G^T \left[\dot{x}_m(t) - F\, x_m(t)\right] \qquad (3\text{-}46)$$

Equation 3-46 represents a feedforward or prefilter type of solution to the model following problem.

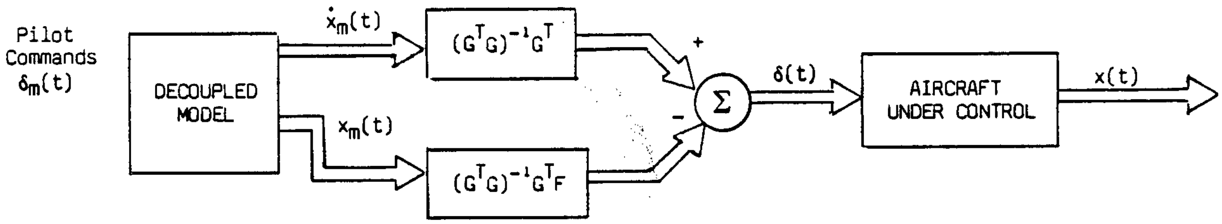


Figure 25. BLOCK DIAGRAM OF EXPLICIT MODEL FOLLOWING CONTROL SYSTEM

The model following system shown above involves only feedforward or a prefilter compensation network and is merely a multidimensional elaboration on the prefilter techniques described in Section 3.3.

The explicit model following technique defined above has been successfully used in the USAF/AFWAL Total In-Flight Simulator (TIFS) for more than 10 years. The decoupling then involves only the equations of motion of a decoupled model. The control system will force the vehicle to fly as defined by the model.

Feedback can be incorporated into the explicit model following technique described above. The model following method used in the TIFS aircraft has been modified to incorporate feedback for the purpose of reducing the sensitivity to knowledge and variation of the stability and control parameters of the TIFS as a function of flight condition.

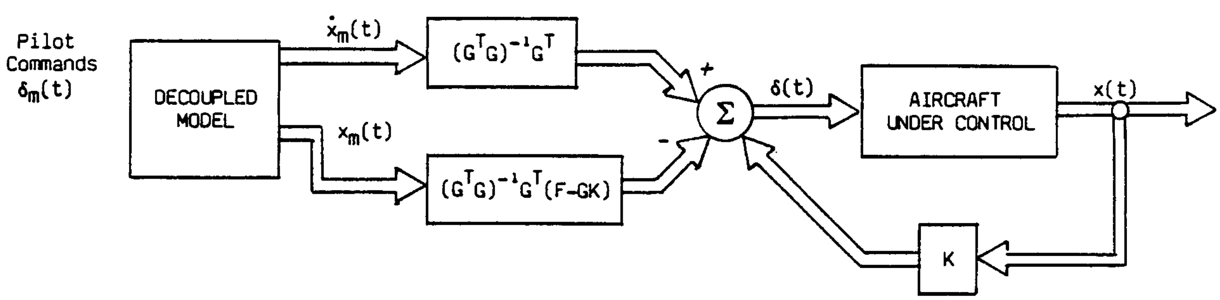


Figure 26. EXPLICIT MODEL FOLLOWING WITH FEEDBACK

From the figure it can be shown that if the feedback gain matrix $K \gg (G^TG)^{-1}G^TF$, the model following system is insensitive to the stability parameters F of the vehicle. The explicit model following method of design then offers a more practical method of achieving a dynamically decoupled flight control system because the "purity" of the decoupling can be controlled and investigated in flight simply by designing impurities or small coupling terms in the model.

3.5 DYNAMIC DECOUPLING (From Reference 8)

Several recent development programs such as the AFTI-16 considered the use of decoupling as a possible way to enhance the maneuver capability or control precision of the flight of the vehicle. The implication in this type of control system design is that the pilot will or must have a separate cockpit manipulator for each degree of freedom of motion of the vehicle that is decoupled from the remaining dynamics of the airframe. Because of this complexity, it seems prudent to investigate very thoroughly before committing to even an experimental program. In fact, a brief investigation of the implications of this method of flight control, given below, can lead the designer to some preliminary conclusions that will guide an experimental investigation.

As an example of dynamic decoupling consider the conventional, two-degree-of-freedom equations of longitudinal-vertical motion for which the airplane is assumed to be flying wings level and at constant speed.

$$\begin{bmatrix} \dot{q} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} M'_q & M'_\alpha \\ 1 & Z_\alpha \end{bmatrix} \begin{bmatrix} q \\ \alpha \end{bmatrix} + \begin{bmatrix} M_{\delta_e} & 0 \\ 0 & Z_{\delta_z} \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_z \end{bmatrix} \qquad (3\text{-}47)$$

where $M'_q = M_q + M_{\dot{\alpha}}$, $M'_\alpha = M_\alpha + Z_\alpha M_{\dot{\alpha}}$, $M'_{\delta_e} = M_{\delta_e} + Z_{\delta_e} M_{\dot{\alpha}}$ and

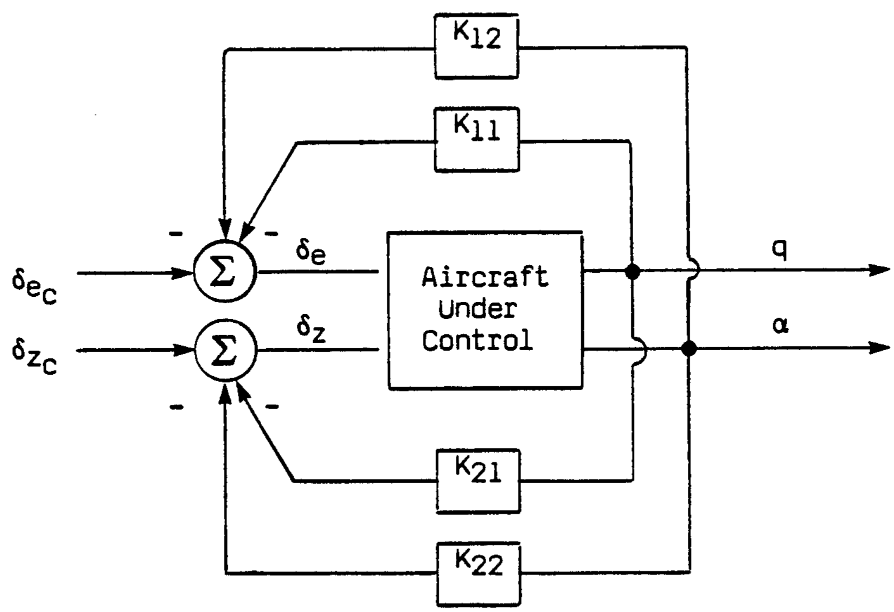
$$n_z = \frac{V}{g}(q - \dot{\alpha})$$

It has been assumed, without loss in generality, that the elevator δ_e produces only a pitching moment while the direct lift device δ_z produces only a normal force, i.e. it acts through the center of gravity of the airplane. In practice, the force and moment effects can be easily accounted for or be cancelled by interconnecting the controllers. Consider the following example.

A very general control law is assumed in which both of the states are fed back to each of the controllers, i.e.

$$\begin{bmatrix} \delta_e \\ \delta_z \end{bmatrix} = -\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} q \\ \alpha \end{bmatrix} + \begin{bmatrix} \delta_{e_c} \\ \delta_{z_c} \end{bmatrix} \tag{3-48}$$

as shown in the sketch given below:



Feedback Control System

Substitution of Equation 3-48 into Equation 3-47 yields the equation of motion for the closed loop aircraft.

$$\begin{bmatrix} \dot{q} \\ \dot{\alpha} \end{bmatrix} = \begin{bmatrix} M'_q & M'_\alpha \\ 1 & Z_\alpha \end{bmatrix} - \begin{bmatrix} M'_{\delta_e} & 0 \\ 0 & Z_{\delta_z} \end{bmatrix} \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} q \\ \alpha \end{bmatrix} + \begin{bmatrix} M_{\delta_e} & 0 \\ 0 & Z_{\delta_z} \end{bmatrix} \begin{bmatrix} \delta_{e_c} \\ \delta_{z_c} \end{bmatrix}$$

$$= \begin{bmatrix} M'_q - K_{11}M_{\delta_e} & M'_\alpha - K_{12}M_{\delta_e} \\ 1 - K_{21}Z_{\delta_z} & Z_\alpha - K_{22}Z_{\delta_z} \end{bmatrix} \begin{bmatrix} q \\ \alpha \end{bmatrix} + \begin{bmatrix} M'_{\delta_e} & 0 \\ 0 & Z_{\delta_z} \end{bmatrix} \begin{bmatrix} \delta_{e_c} \\ \delta_{z_c} \end{bmatrix} \tag{3-49}$$

$$n_z = \frac{V}{g}(q - \dot{\alpha})$$

It can be seen that full state feedback, as represented by Equation 3-49 results in the ability to independently augment each of the stability derivatives. The augmented stability derivative $M'_\alpha - K_{12}M'_{\delta_e}$ represents the dynamic coupling of the Z force equation into the pitching moment equation and the term $1 - K_{21}Z_{\delta_z}$ represents the coupling of the pitching moment equation into the Z force equation.

The transfer functions for $q(s)$ and $\alpha(s)$ to the command inputs $\delta_{e_c}(s)$ and $\delta_{z_c}(s)$ are given by

$$q/\delta_{e_c}(s) = \frac{M'_{\delta_e}(s - Z_\alpha + K_{22}\, Z_{\delta_z})}{\Delta(s)} \tag{3-50}$$

$$\frac{\alpha}{\delta_{e_c}}(s) = \frac{M'_{\delta_e}(1 - K_{21}\, Z_{\delta_z})}{\Delta(s)} \tag{3-51}$$

$$\frac{q}{\delta_{z_c}}(s) = \frac{Z_{\delta_z}(M'_\alpha - K_{12}\, M'_{\delta_e})}{\Delta(s)} \tag{3-52}$$

$$\frac{\alpha}{\delta_{z_c}}(s) = \frac{Z_{\delta_z}(s - M'_q + K_{11}\, M'_{\delta_e})}{\Delta(s)} \tag{3-53}$$

and $\Delta(s) = (s - M'_q + K_{11}M'_{\delta_e})(s - Z_\alpha + K_{22}Z_{\delta_z}) + (1 - K_{21}Z_{\delta_z})(-M'_\alpha + K_{12}M'_{\delta_e})$

(3-54)

In Equation 3-54 $(s - M'_q + K_{11}M'_{\delta_e})$ represents the augmented pitching mode and $(s - Z_\alpha + K_{22}Z_{\delta_z})$ represents the augmented heave mode of motion.

As shown by Equation 3-51 if the coupling term $1 - K_{21}Z_{\delta_z}$ is zero, the transfer function $\alpha/\delta_{e_c}(s)$ is zero and $\alpha/\delta_{e_c}(s)$ is decoupled from a $\delta_{e_c}(s)$ input. Similarly, if $M_\alpha - K_{12}M_{\delta_e}$ is zero, then q is decoupled from a δ_{z_c} input.

Dynamic decoupling of the pitch and vertical translation degrees of freedom of motion shows that the responses in q and α will be first order, i.e., the variables will respond in their pure modes of motion. Notice that decoupling using feedback can only be accomplished by feedback from the decoupled state to a separate controller, i.e. decoupling of angle of attack from the δ_{e_c} command is accomplished by feedback from pitch rate to the δ_z controller. Similarly, decoupling of q from a δ_{z_c} command is accomplished by feedback from α to the δ_e input.

Consider angle of attack decoupled from a δ_{e_c} input and q decoupled from a δ_{z_c} input. From Equations 3-51 and 3-52,

$$\frac{\alpha}{\delta_{e_c}}(s) = \frac{q}{\delta_{z_c}}(s) = 0 \quad \text{iff}$$

$$1 - K_{21}Z_{\delta_z} = 0 \; ; \qquad K_{21} = 1/Z_{\delta_z}$$

$$M'_\alpha - K_{12}M'_{\delta_e} = 0 \; ; \qquad K_{12} = \frac{M'_\alpha}{M'_{\delta_e}}$$

Then

$$\frac{q}{\delta_{e_c}}(s) = \frac{M'_{\delta_e}(s - Z_\alpha + K_{22}Z_{\delta_z})}{(s - M'_q + K_{11}M'_{\delta_e})(s - Z_\alpha + K_{22}Z_{\delta_z})} = \frac{M'_{\delta_e}}{s - M'_q + K_{11}M'_{\delta_e}} \tag{3-55}$$

$$\frac{\alpha}{\delta_{z_c}}(s) = \frac{Z_{\delta_z}(s - M_q' + K_{11}M_{\delta_e}')}{(s - M_q' + K_{11}M_{\delta_e}')(s - Z_\alpha + K_{22}Z_{\delta_z})} = \frac{Z_{\delta_z}}{s - Z_\alpha + K_{22}Z_{\delta_z}} \tag{3-56}$$

The vertical acceleration responses become

$$\frac{n_z}{\delta_{e_c}} = \frac{V}{g}\,\frac{q}{\delta_{e_c}}(s) \quad \text{and} \quad \frac{n_z}{\delta_{z_c}} = \frac{V}{g}\,\frac{\dot{\alpha}}{\delta_{z_c}}(s) \tag{3-57}$$

The requirements for the direct lift mode of response are easily obtained from Equation 3-52

$$\frac{n_z}{\delta_{z_c}}(s) = \frac{V}{g}\,\frac{\dot{\alpha}}{\delta_{z_c}}(s) = \frac{V}{g}\,\frac{sZ_{\delta_z}}{(s - Z_\alpha + K_{22}Z_{\delta_z})} \tag{3-58}$$

$$\text{If } K_{22} = \frac{Z_\alpha}{Z_{\delta_z}}, \text{ then } \frac{n_z}{\delta_{z_c}}(s) = \frac{V}{g} Z_{\delta_z}, \tag{3-59}$$

the direct lift command mode. The direct lift command mode, therefore, requires feedback that has the effect of cancelling the natural Z_α of the aircraft.

There is reason to believe that the decoupling of pitch from angle of attack can be useful because, from Equation 3-57

$$\frac{n_z}{\delta_{e_c}}(s) = \frac{V}{g}\,\frac{q}{\delta_{e_c}}(s) \quad \text{or} \quad \frac{\dot{\gamma}}{\delta_{e_c}}(s) = \frac{q}{\delta_{e_c}}(s)$$

When pitch rate is decoupled from angle of attack, the flight path change and the pitch angle change are identical; the change in pitch angle is perpendicular to the radius of curvature during a maneuver and the aircraft is heading in the direction it is pointed, even during the short term duration of the vehicle response.

3.6 PARTIAL DECOUPLING

It is a relatively straightforward calculation to define the feedback required to either partially or fully decouple an airplane. Consider the three-degree-of-freedom equations of longitudinal-vertical motion:

$$\begin{bmatrix} \dot{u} \\ \dot{\theta} \\ \dot{q} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} X_u & -g & 0 & X_w \\ 0 & 0 & 1 & 0 \\ M_u & 0 & M'_q & M'_w \\ Z_u & 0 & u_0 & Z_w \end{bmatrix} \begin{bmatrix} u \\ \theta \\ q \\ w \end{bmatrix} + \begin{bmatrix} X_{\delta_T} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & M'_{\delta_e} & 0 \\ 0 & 0 & Z_{\delta_z} \end{bmatrix} \begin{bmatrix} \delta_T \\ \delta_e \\ \delta_z \end{bmatrix} \qquad (3\text{-}60)$$

or, in general, the following matrix vector set can represent these equations of motion:

$$\dot{x} = Fx + G\delta$$

when $x^T = [u, \theta, q, w]$, the state vector, and $\delta^T = [\delta_T, \delta_e, \delta_z]$ is the control vector.

The above equation represents the three degrees of freedom of motion of an airplane and it is assumed that three independent controllers, δ_T = throttle or X force device, δ_e = elevator or pitching moment device and δ_z = direct lift effector, are available for feedback control. The dynamic system is completely controllable and can be completely decoupled or made to respond in almost any way desired.

Assume that it is desirable to decouple the vertical velocity response of the vehicle from pitching motions or from speed changes. To do this, it is only necessary to feedback such that the augmented dynamics have null entries in elements 4, 1 and 4, 3 of the equations of motion. It is only necessary to feedback speed change and pitch rate to the direct lift effector, as

$$\delta_z = -K_{13}u - K_{33}q + \delta_{z_c} \qquad (3\text{-}61)$$

Substitution of this control law into Equation 3-60 yields the closed loop equations of motion:

$$\begin{bmatrix} \dot{u} \\ \dot{\theta} \\ \dot{q} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} X_u & -g & 0 & X_w \\ 0 & 0 & 1 & 0 \\ M'_u & 0 & M'_q & M'_w \\ Z_u - K_{13}Z_{\delta_z} & 0 & U_o - K_{33}Z_{\delta_z} & Z_w \end{bmatrix} \begin{bmatrix} u \\ \theta \\ q \\ w \end{bmatrix} + \begin{bmatrix} X_{\delta_T} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & M'_{\delta_e} & 0 \\ 0 & 0 & Z_{\delta_z} \end{bmatrix} \begin{bmatrix} \delta_T \\ \delta_e \\ \delta_{z_c} \end{bmatrix} \tag{3-62}$$

The values of the feedback gains K_{13} and K_{33} are obtained simply from

$$Z_u - K_{13}Z_{\delta_z} = 0 \; , \quad K_{13} = Z_u/Z_{\delta_z}$$

$$U_o - K_{33}Z_{\delta_z} = 0 \; , \quad K_{33} = U_o/Z_{\delta_z}$$

The closed loop equations of motion then are:

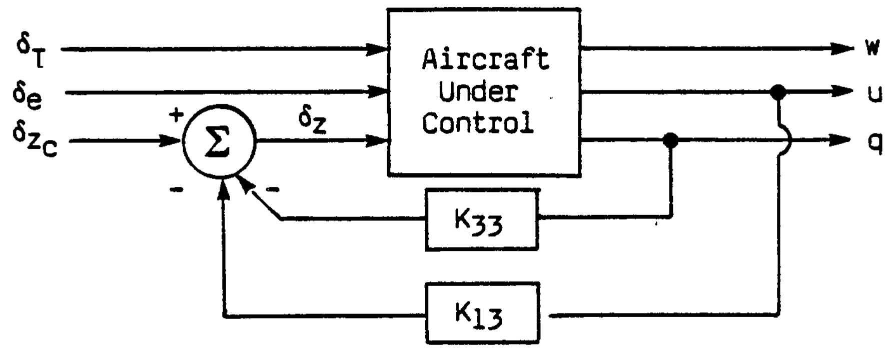
$$\begin{bmatrix} \dot{u} \\ \dot{\theta} \\ \dot{q} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} X_u & -g & 0 & X_w \\ 0 & 0 & 1 & 0 \\ M_u & 0 & M'_q & M'_w \\ 0 & 0 & 0 & Z_w \end{bmatrix} \begin{bmatrix} u \\ \theta \\ q \\ w \end{bmatrix} + \begin{bmatrix} X_{\delta_T} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & M'_{\delta_e} & 0 \\ 0 & 0 & Z_{\delta_z} \end{bmatrix} \begin{bmatrix} \delta_T \\ \delta_e \\ \delta_{z_c} \end{bmatrix} \tag{3-63}$$

The last equation of the matrix set is now decoupled, yielding the equation for vertical velocity

$$\dot{w} = Z_w w + Z_{\delta_z} \delta_{z_c} \quad \text{or} \quad \frac{w}{\delta_{z_c}}(s) = \frac{Z_{\delta_z}}{s - Z_w} \tag{3-64}$$

and speed changes or pitching motion will not affect the vertical velocity response. The converse is not true.

In block diagram form, the system that decouples the vertical velocity from speed changes and pitching motions is shown in the sketch below:



Vertical Velocity Decoupled from Pitch and Speed Change

As a second example, consider the relatively simple problem of decoupling speed change from pitching and vertical velocity changes. The control law is simply

$$\delta_T = -K_{12}\theta - K_{14}w + \delta_{T_c} \tag{3-65}$$

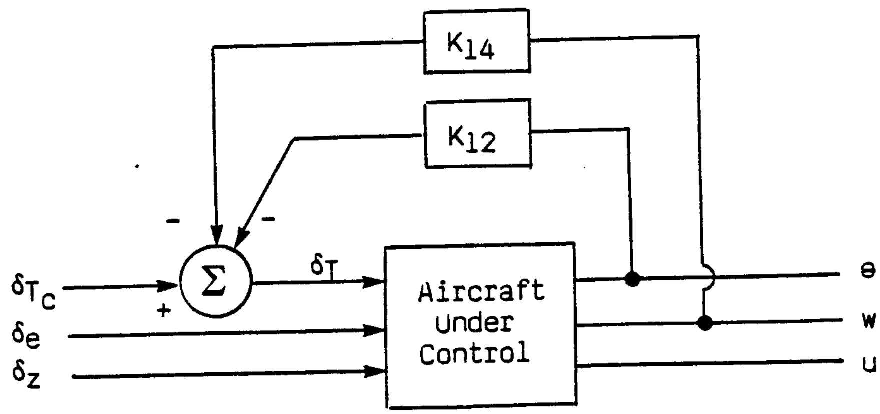
where

$$K_{12} = -\frac{-g}{x_{\delta_T}} \quad \text{and} \quad K_{14} = \frac{x_w}{x_{\delta_T}}$$

The response of the change in speed is given by

$$\dot{u} = x_u u + x_{\delta_T}\delta_{T_c} \quad \text{or} \quad \frac{u}{\delta_{T_c}}(s) = \frac{x_{\delta_T}}{s-x_u} \;. \tag{3-66}$$

In block diagram form, the system is shown in the sketch below:



Speed Change Decoupled from Pitch and Vertical Velocity

In general, a feedback control law is defined as

$$u = -Kx$$

and a control law that decouples each of the states from the others is given by

$$\begin{bmatrix} \delta_T \\ \delta_e \\ \delta_z \end{bmatrix} = -\begin{bmatrix} 0 & K_{12} & 0 & K_{14} \\ K_{21} & 0 & 0 & K_{24} \\ K_{31} & 0 & K_{33} & 0 \end{bmatrix}\begin{bmatrix} u \\ \theta \\ q \\ w \end{bmatrix} + \begin{bmatrix} x_{\delta_T} & 0 & 0 \\ 0 & M'_{\delta_e} & 0 \\ 0 & 0 & Z_{\delta_z} \end{bmatrix}\begin{bmatrix} \delta_{T_c} \\ \delta_{e_c} \\ \delta_{z_c} \end{bmatrix} \tag{3-67}$$

and the closed loop system description becomes:

$$\begin{bmatrix} \dot{u} \\ \dot{\theta} \\ \dot{q} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} x_u & -g-K_{12}x_{\delta_T} & 0 & x_w-K_{14}x_{\delta_T} \\ 0 & 0 & 1 & 0 \\ M'_u-K_{21}M'_{\delta_e} & 0 & M'_q & M'_w-K_{24}M'_{\delta_e} \\ Z_u-K_{31}Z_{\delta_z} & 0 & U_0-K_{33}Z_{\delta_z} & Z_w \end{bmatrix}\begin{bmatrix} u \\ \theta \\ q \\ w \end{bmatrix} + \begin{bmatrix} x_{\delta_T} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & M'_{\delta_e} & 0 \\ 0 & 0 & Z_{\delta_z} \end{bmatrix}\begin{bmatrix} \delta_{T_c} \\ \delta_{e_c} \\ \delta_{z_c} \end{bmatrix} \tag{3-68}$$

If, in addition to the feedback gains defined in Equation 3-65 the following feedback is added,

$$\delta_c = -K_{21}u - K_{24}w + \delta_{e_c}$$

where

$$K_{21} = \frac{M'_u}{M'_{\delta_e}} \quad \text{and} \quad K_{24} = \frac{M'_w}{M'_{\delta_e}}$$

the system is completely decoupled dynamically, with closed loop equations of motion

$$\begin{bmatrix} \dot{u} \\ \dot{\theta} \\ \dot{q} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} X_u & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & M'_q & 0 \\ 0 & 0 & 0 & Z_w \end{bmatrix} \begin{bmatrix} u \\ \theta \\ q \\ w \end{bmatrix} + \begin{bmatrix} X_{\delta_T} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & M'_{\delta_e} & 0 \\ 0 & 0 & Z_{\delta_z} \end{bmatrix} \begin{bmatrix} \delta_{T_c} \\ \delta_{e_c} \\ \delta_{z_c} \end{bmatrix} \tag{3-69}$$

and reduces simply to the three first order equations

$$\dot{u} = X_u u + X_{\delta_T} \delta_{T_c}$$

$$\dot{q} = M_q q + M'_{\delta_e} \delta_{e_c} \quad \text{or} \quad \frac{\theta}{\delta_{e_c}}(s) = \frac{M'_{\delta_e}}{s(s - M_q)}$$

$$\dot{w} = Z_w w + Z_{\delta_z} \delta_{z_c}$$

The time constants of the first order responses of the decoupled system can be changed by adding the feedback $u \rightarrow \delta_T$, $q \rightarrow \delta_e$ and $w \rightarrow \delta_z$ to the control law.

In general, then, to decouple a response variable from the other responses of the system requires feedback from the response variables other than the decoupled response to the controller used to excite the decoupled response variable. In other words, to decouple w from q and u required feedback from u and q to δ_z; in shorthand, u, q $\rightarrow \delta_z$.

The decoupling feedback gains can be sensitive to the accuracy of the stability and control derivatives used to calculate the feedback gains. An obvious way to reduce this sensitivity is to decrease the time constant of the decoupled response; i.e. by regulating the primary response.

For instance, consider the pitching moment equation

$$\dot{q} = M_u'u + M_q'q + M_w'w + M_{\delta_e}' \delta_e$$

If the control law is $\delta_e = -K_{23}q + \delta_{e_c}$, then the above equation becomes

$$\dot{q} = M_u u + (M_q' - K_{23}M_{\delta_e}')q + M_w'w + M_{\delta_e}'\delta_e \qquad (3\text{-}70)$$

and if $M_q' - K_{23}M_{\delta_e}'$ is very, very large, then

$$\dot{q} \approx (M_q - K_{23}M_{\delta_e}')q + M_{\delta_e}'\delta_{e_c} \qquad (3\text{-}71)$$

and the pitching moment equation approaches decoupling. The pitch response is then insensitive to the values of $M_u'u$ and $M_w'w$. In fact, if $M_q' - K_{23} M_{\delta_e}' \to \infty$, i.e., if $K_{23} \to \infty$, then $q \approx 1/K_{23}\ \delta_{e_c} \approx 0$ and pitch does not respond to a δ_{e_c} input.

3.7 GUST BEHAVIOR

The primary reason for a flight control sytem is to provide for satisfactory and acceptable flying qualities. There are, however, a number of normally secondary flight control system design objectives, including gust alleviation and possibly structural mode control.

In general, satisfactory flying qualities can be obtained either using feedback or using feedforward/command augmentation. In theory, gust alleviation can also be accomplished using feedback or, if the turbulence is directly measurable, using an open loop alleviation method. This is true only if the augmented airplane is stable. Therefore, basic stability, either of a rigid body mode or structural mode, can be obtained only using feedback.

The usual approach to gust alleviation in the past has been to feedback normal acceleration to the pitching moment surface or to then direct lift surface. The purpose is to regulate the vehicle to gust inputs as tightly as possible. The command input gain is increased to allow the pilot to maneuver the airplane. This approach, although sometimes effective, has a few inherent problems as well. For instance:

A tight regulator design may decrease the sensitivity of the vehicle to turbulence at the low frequency end of the spectrum, but will likely extend the bandwidth of the closed loop response, thereby increasing the sensitivity to high frequency turbulence inputs, particularly in the frequency range (1-3 Hz) that would likely be objectionable to the pilot.

Feedback for gust alleviation must be inertial feedback; pitch rate, normal acceleration and inertial angle of attack are among the logical choices, but feedback from an angle of attack vane is generally unacceptable. To illustrate, consider the simple two degree of freedom equation of motions of an aircraft with neutral static stability, defined as $M_\alpha = 0$. It is also assumed that $Z_{\delta_e} \doteq 0$ and a uniform gust field is encountered; then

$$\begin{bmatrix} \dot{q}(t) \\ \dot{\alpha}_I(t) \end{bmatrix} = \begin{bmatrix} M_q + M_\alpha & M_{\dot{\alpha}} Z_\alpha \\ 1 & Z_\alpha \end{bmatrix} \begin{bmatrix} q(t) \\ \alpha_I(t) \end{bmatrix} + \begin{bmatrix} M_{\delta_e} \\ 0 \end{bmatrix} \delta_e(t) + \begin{bmatrix} M_{\dot{\alpha}} Z_\alpha \\ Z_\alpha \end{bmatrix} \alpha_g(t) \tag{3-72}$$

$$n_z(t) = \frac{V}{g}(q - \dot{\alpha}) = + \frac{V}{g} Z_\alpha \alpha(t)$$

Assume feedback from a vane that senses $\alpha_I + \alpha_g$ to the elevator; i.e., $\delta_e = \delta_{e_c} - K(\alpha_I + \alpha_g)$. The closed loop equations of motion become

$$\begin{bmatrix} \dot{q}(t) \\ \dot{\alpha}_I(t) \end{bmatrix} = \begin{bmatrix} M_q + M_{\dot{\alpha}} & M_{\dot{\alpha}} Z_\alpha + K M_{\delta_e} \\ 1 & Z_\alpha \end{bmatrix} \begin{bmatrix} q(t) \\ \alpha_I(t) \end{bmatrix} + \begin{bmatrix} M_{\delta_e} \\ 0 \end{bmatrix} \delta_e + \begin{bmatrix} M_{\dot{\alpha}} Z_\alpha + K M_{\delta_e} \\ Z_\alpha \end{bmatrix} \alpha_g \tag{3-73}$$

which shows that the steady state or low frequency amplitude of the $\alpha(t)$ response to turbulence is unchanged but the response to a command input gearing is reduced and the frequency band of the response has been extended In short, nothing positive has been done for gust alleviation, either with respect to the angle of attack response or vertical acceleration response.

If, however, the feedback gain is selected such that $M_{\dot{\alpha}} Z_{\alpha} + K\, M_{\delta_e} = 0$; i.e., $K = M_{\dot{\alpha}}\, Z_{\alpha} / M_{\delta_e}$, then not only has the turbulence input to the pitching moment equation been eliminated, but the pitching moment equation has been decoupled from the vertical force equation. In effect, gust alleviation using feedback can be accomplished either by designing a tight regulator or by decoupling the system; i.e., by regulating or deregulating.

Feedback from a sensed inertial angle of attack or pitch rate gyro affects only the denominator of the transfer function of the vehicle responses to turbulence. Assuming that the feedback is in the sense to increase the bandwidth of the response, the sensitivity of the response to turbulence will be decreased at the low frequency end of the spectrum as the bandwidth is extended.

A direct measurement of gust angle of attack can theoretically be used to gust alleviate the airplane if the gust inputs are used to drive both an elevator and a direct lift flap (Reference 9). The equation of motion are of the form

$$\dot{x}(t) = F\, x(t) + G\, u(t) + J\, \alpha_g(t) \tag{3-74}$$

If u(t) is chosen such that $u(t) = -G^{-1} J\, \alpha_g(t)$ then the control surfaces are driven to produce forces and moments on the airplane that exactly counter the forces and moments produced on the airplane by the gusts. If G and J are defined:

$$G = \begin{bmatrix} M_{\delta_e} & M_{\delta_z} \\ Z_{\delta_e} & Z_{\delta_z} \end{bmatrix} \qquad J = \begin{bmatrix} M_{\dot{\alpha}}\, Z_{\alpha} \\ Z_{\alpha} \end{bmatrix}$$

Then the control law for "exact" gust alleviation is

$$\begin{bmatrix} \delta_e \\ \delta_z \end{bmatrix} = - \frac{1}{M_{\delta_e} Z_{\delta_z} - M_{\delta_z} Z_{\delta_e}} \begin{bmatrix} Z_{\delta_z} & -M_{\delta_z} \\ -Z_{\delta_e} & M_{\delta_e} \end{bmatrix} \begin{bmatrix} M_{\dot{\alpha}} Z_{\alpha} \\ Z_{\alpha} \end{bmatrix} \alpha_g$$

$$\begin{bmatrix} \delta_e \\ \delta_z \end{bmatrix} = - \frac{1}{M_{\delta_e} Z_{\delta_z} - M_{\delta_z} Z_{\delta_e}} \begin{bmatrix} Z_{\delta_z} M_{\dot{\alpha}} Z_{\alpha} - M_{\delta_z} Z_{\alpha} \\ -Z_{\delta_e} M_{\dot{\alpha}} Z_{\alpha} + M_{\delta_e} Z_{\alpha} \end{bmatrix} \alpha_g \tag{3-75}$$

$$= \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \alpha_g$$

The gust alleviation then consists of a measurement of the gust-induced angle of attack multiplied by gains and fed into the elevator and direct lift flaps. The alleviation is entirely open loop if the gusts are measured accurately and theoretically "perfect" if the number of available controllers is equal to the number of coupled degrees of freedom of motion.

Section 4
CONCLUSIONS AND RECOMMENDATIONS

1. The large majority of the experimental data used to specify the modal requirements in MIL-F-8785B(ASG) were obtained using "conventional" or angle of attack command aircraft. The proposed MIL-F-8785(C) handbook does not demonstrate that data collected to define flying qualities requirements for "conventional" or angle of attack command aircraft can be applied directly to pitch rate command system.

   - Recommendation

     Flight experiments be performed to determine the extent to which requirements for angle of attack command systems can be applied to pitch rate command systems.

2. A system is pitch rate command or angle of attack command depending upon the location of the poles relative to zeros of the transfer functions. A system can be pitch rate command relative to the short period poles and angle of attack command relative to the phugoid dynamics or vice versa.

   - Recommendation

     Flight experiments be devised to determine pilot preference with respect to pitch rate and angle of attack command, both with respect to the short period and the phugoid modes of response of the vehicle.

3. Flying qualities requirements are at best remotely related to a particular conceptual control system configuration or architecture. Identical dynamic behavior to pilot command inputs can be obtained using a large variety of feedback and prefilter configurations.

   - Recommendation

     Flying qualities criteria be investigated independently of control system architecture. Flight experiments be designed to demonstrate that nearly identical responses to pilot commands can be obtained using either feedback or prefilters.

4. Because a prefilter or command augmentation system can be used to alter the dynamic behavior of a system to a command input without changing the feedback, a variety of prefilters can be designed for particular or specialized flight tasks.

   - Recommendation

     It is recommended that a prefilter be designed to change an angle of attack command system into a pitch rate command/attitude hold system. The results should be compared with a feedback system designed for pitch rate command/attitude hold.

5. It appears that several dynamically decoupled situations could be beneficial to flight precision, while others may offer little in the way of improved performance, partly because of the requirements for separate cockpit controllers and partly because of the required control system complexity required.

   - Recommendation

     It is recommended that a comprehensive study of decoupling flight control system configurations be performed on a moving base ground simulator before committing to an experimental flight test program.

Section 5

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Appendix A
TASK PLAN

A.1 BACKGROUND

Recent experimental flight test results for the approach and landing phase of flight strongly suggest that at least five main factors should be considered in the specification of flying qualities to guide a flight control system designer. These are:

1. Short Term Response

   The short term angle of attack response of the vehicle is defined by the ω_n vs n/α item in the flying qualities specification MIL-F8785 (C). The short period angle of attack response is chosen as having been defined by this specification because the two degree of freedom transfer function of the angle of attack response contain only short period natural frequency and damping ratio as dynamic parameters.

2. Time Delay in the $\dot{q}(t)$ or $q(t)$ Response

   This item addresses the allowable higher order lags, transport delays and non-minimum phase characteristics that may be introduced by the control system mechanization.

3. Pilot Location With Respect to Rotation Center

   This requirement should be used in association with the short period frequency requirement of the specification and addresses two factors: (1), the delay or quickening resulting from pilot location can be interpreted as an equivalent change in short period frequency and (2), a non-minimum phase response would produce an initial pilot cue that the aircraft was responding in an improper direction.

4. Flight Path Response Delay

The ultimate result of a pilot input is almost always a change in flight path. The flight path response to pilot commands is almost always non-minimum phase. This non-minimum phase response can likely be treated as a delay parameter. The limits of this delay parameter should be established, particularly for precision control tasks such as approach and landing or in-flight refueling.

5. Long Term or Phugoid-Like Behavior

The limits of the long term behavior of the vehicle response should be established in terms of allowable residues of the low frequency poles for the response variables of the airplane. Specifically, since $\gamma = \theta - \alpha$, a pitch rate command, attitude hold system implies $\Delta\gamma = -\Delta\alpha$ after a new attitude has been "held" in the rate command, attitude hold system. An angle of attack command system has minimum residue in the low frequency mode in the angle of attack response, so after the short term response $\dot{\alpha}(t) = 0$, and $\Delta\gamma = \Delta\theta$. The result is that if angle of attack is changing following the short term response, the pilot cannot use changes in pitch attitude as an indicator of changes in flight path angle. It is likely that a pilot will require a direct flight path display device such as a HUD to fly a precision flight path with a pitch rate command, attitude hold system. On the other hand, an angle of attack command system will yield $\Delta\theta = \Delta\gamma$ and the pilot may then use changes in attitude as a direct indicator of change in flight path.

The problem of trying to define acceptable flying qualities specifications for both pitch and vertical velocity degrees of freedom involves a definition of task, short period behavior and long term behavior. If the task is maneuvering, such as approach and landing, then the major emphasis may likely be on flight path definitiion with a minor limitation of defining allowable pitching motion with respect to the flight path specifications. If the task is strictly pitch oriented, such as photography with respect to a body-fixed camera or a laser weapon, then it is possible that precision attitude control without respect to flight path is most acceptable. There is room to believe that an angle of attack control system, in which pitch angle changes are equal to flight path angle changes, i.e., the airplane goes in the direction it is pointed, would be even more acceptable. These are the kinds of questions to be addressed in the proposed flying qualities/flight control program.

A.2 EXPERIMENTAL OBJECTIVES

The general objective of the proposed experiment is to link or associate flying qualities requirements to the control system design concept or methodology. By establishing this link, the following is addressed:

1. The relationship between dynamic behavior of specific response variables of the airplane and particular items of MIL-F-8785(C). This in turn establishes a pole-zero relationship or residue relationship of particular modes of motion with respect to the vehicle response variables.

2. The relationship between the short term and long term longitudinal response requirements. For instance, whether or not the short term response can be considered separately in terms of a flying qualities analysis depends upon the frequency separation between phugoid and short period as well as the residues of the short term and long term modes in particular responses of the airplane.

3. The response variables of major importance for a particular task; in this case approach and landing.

A problem with some of the flying qualities experiments previously performed has been a lack of attention to excluding other factors that may influence the results. In this case, the objective will be to try to maintain a constant position of the pilot with respect to the center of gravity of the vehicle. Specifically, the pilot will be located on the center of rotation so that his normal acceleration cues will be due entirely to flight path rotation. The center of gravity will be located such that the non-minimum phase "time delay" in the $\gamma(t)$ response is constant and approximately 0.60 seconds, (apparently within the level 1 requirements). Speed change dynamics will be entirely defined by the phugoid mode. The speed change response will be as invariant as possible throughout the experiment, changing only as the phugoid dynamics change.

The experiment will then be set up to exclusively investigate residues in different responses of two degrees of freedom, pitch and vertical velocity and in the two primary modes of motion, short term or short period and phugoid. Other influencing variables, such as direct lift effects, speed change characteristics, and pitch and flight path time delays will be held constant to the extent possible.

There will be no attempt to deliberately introduce higher order dynamics, such as proportional plus integral compensation into the picture. These are artifacts of the mechanization that can strongly influence the resulting dynamic picture, and will be excluded <u>in this first stage of experimentation</u>.

A second stage of the experiment should investigate differences in vehicle behavior as a function of how the system is mechanized. For instance, if it is assumed that the vehicle is statically unstable and pitch rate is to be the only sensor, then that sensor must be used for feedback to obtain stability. Flying qualities requirements beyond the requirements to obtain basic stability can be obtained using either feedback compensation or command augmentation, i.e., prefilters. Both mechanizations can yield identical responses to pilot commands inputs, but they will behave differently in turbulence and have different phase and gain margins with respect to aeroservoelastic effects.

A successful completion of the flying qualities experiment proposed above should establish a solid basis for the interpretation of flying qualities requirements for use by the flight control system design engineer.

A.3 PROPOSED CONFIGURATION MATRIX

1. Modally Decoupled Configurations

These configurations are designed to establish pilot acceptance or rejection of pitch rate or angle of attack command systems for both short period and phugoid or long term modes of response. Two important variations in dynamics are designed into the experiment. These are $1/\tau_{\theta_2}$ variations of $1/\tau_{\theta_2} = 0.5$ and $1/\tau_{\theta_2} = 0.9$. The second is a variation of the phugoid frequency from $\omega_p = 0.2$ to $\omega_p = 0.1$. Both of these variations very significantly affect the character or signature of the response variables to a pilot command input. These eight configurations are listed in Table 1 of the report, repeated here for convenience.

Table 1
RATE AND PATH COMMAND CONFIGURATIONS

SYSTEM	FIG	CONFIG.	ω_{sp}(rad/sec)	ζ_{sp}	ω_{ph}(rad/sec)	ζ_{ph}	$1/\tau_{\theta_2}$
1	6	I-A	$p_1 = -.500$	$p_2 = -8.0$	$p_3 = -0.10$	$p_4 = 0.0$	0.50
2	7	I-B	2.00	0.70	0.20	0.10	0.50
3	8	I-C	2.00	0.70	$p_3 = -0.10$	$p_4 = 0.0$	0.50
4	9	I-D	$p_1 = -.500$	$p_2 = -8.00$	0.20	0.10	0.50
1	10	II-A	$p_1 = -0.90$	$p_2 = -4.40$	$p_3 = -0.10$	$p_4 = 0.0$	0.90
2	11	II-B	2.00	0.70	0.10	0.10	0.90
3	12	II-C	2.00	0.70	$p_3 = -0.10$	$p_4 = 0.0$	0.90
4	13	II-D	$p_1 = -0.90$	$p_2 = -4.40$	0.10	0.10	0.90

Note: $1/\tau_{\theta_1} = 0.10$ for all cases

2. Dynamic Docoupling

Two dynamic decoupling configurations are added to round out the proposed experimental program.

1) Angle of attack decoupled from stick command

This configuration is defined simply by the transfer function for pitch rate and for velocity change:

$$\frac{q}{\delta_c}(s) = \frac{-20}{s + 4.4} \qquad (1/\tau_{\theta_2} = 0.9)$$

$$\frac{\Delta V}{\delta_c}(s) = \frac{-25(s + 1)(s - 15)}{s(s + 0.10)(s + 0.9)(s + 4.4)}$$

$$\frac{\alpha}{\delta_c}(s) = 0$$

2) Pitch rate decoupled from stick command

$$\frac{q}{\delta_{z_c}}(s) = 0$$

$$\frac{\alpha}{\delta_{z_c}}(s) = \frac{Z_{\delta_z}}{s - Z_\alpha} = \frac{-.20}{s + 0.9}$$

$$\frac{\Delta V}{\delta_{z_c}}(s) = \frac{K(s + 1)(s - 15)}{s(s + 0.10)(s + 0.9)(s + 4.4)}$$

where K will be selected to be representative of the TIFS airplane.

3) As given in the main body of the report, the equation for decoupling and intermediate coupling are given by

$$1 - K_{21} Z_{\delta_z} = 0$$

$$M_\alpha - K_{12} M_{\delta_e} = 0$$

when K_{12} represents feedback of angle of attack to the elevator and K_{21} represents pitch rate feedback to the direct lift flap. The degree of coupling is represented by how closely the equations given above are satisfied. It is reccommended that Z_{δ_z} and M_{δ_e} be changed by 20% without altering the values of K_{12} and K_{21}. This tolerance of 20% represents a normal degree of uncertainty in the knowledge of Z_{δ_z} and M_{δ_e} common in aircraft. The variation suggested will enable the designer to determine whether or not the degree of "impurity" that can be expected in a dynamically decoupled system will affect the flying qualities.